# CONTINUITY & DIFFERENTIABILITY

## EXERCISE – I HINTS & SOLUTIONS

**Sol.1 A**

$$f(x) = \begin{cases} \dfrac{\cos(\sin x) - \cos x}{x^2}, & x \neq 0 \\ a, & x = 0 \end{cases}$$

for continuity, $\lim\limits_{x \to 0} f(x) = f(0)$

$$\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^2} = a$$

$$\lim_{x \to 0} \frac{2\sin\left(\frac{x - \sin x}{2}\right)\sin\left(\frac{\sin x + x}{2}\right)}{x^2} = a$$

$$\lim_{x \to 0} \frac{2\sin\left(\frac{x - \sin x}{2}\right)}{\left(\frac{x - \sin x}{2}\right)} \times \frac{\sin\left(\frac{x + \sin x}{2}\right)}{\left(\frac{x + \sin x}{2}\right)} \times \left(\frac{x^2 - \sin^2 x}{4}\right) = a$$

$$2 \times \frac{0}{4} = a \Rightarrow a = 0$$

**Sol.2 B**

$$f(x) = \begin{cases} \dfrac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \le x \le 0 \\ \dfrac{2x+1}{x+2}, & 0 \le x \le 1 \end{cases}$$

since it is cont, so,

$$\lim_{x \to 0^-} f(x) = f(0)$$

$$\lim_{h \to 0} \frac{\sqrt{1+p(-h)} - \sqrt{1-p(-h)}}{-h} = -\frac{1}{2}$$

$$\lim_{h \to 0} \frac{(1-ph) - (1+ph)}{-h\left\{\sqrt{1-ph} + \sqrt{1+ph}\right\}} = -\frac{1}{2}$$

$$\frac{+2p}{2} = -\frac{1}{2}$$

$$p = -1/2$$

**Sol.3 D**

$$f(x) = \left|\left(x + \frac{1}{x}\right)[x]\right|, \quad x \in [-2, 2]$$

$$\lim_{x \to 2^-} f(x) = \lim_{h \to 0} \left|\left(\frac{5}{2} - h\right)1\right| = \frac{5}{2}$$

$$f(2) = \left|\frac{5}{2} \times 2\right| = 5$$

so, disc at x = 2

now defining function

$$f(x) = \left|\left(x + \frac{1}{2}\right)[x]\right| = \begin{cases} 3 & ; \ -2 \le x < -1 \\ \frac{1}{2} & ; \ -1 \le x < 0 \\ 0 & ; \ 0 \le x < 1 \\ \frac{3}{2} & ; \ 1 \le x \le 2 \\ 2 & ; \ 2 \le x < 3 \end{cases}$$

by defining the function we can say that this it is disc at x = 0

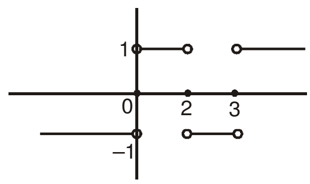
**Sol.4 C**

$f(x) = \text{sgn}(x)$, $g(x) = x(x^2 - 5x + 6)$

$f(g(x)) = \text{sgn}(x(x^2 - 5x + 6))$

$\quad = \text{sgn}(x(x-2)(x-3))$

$$g(g(x)) = \begin{cases} 1 & ; \ x(x-2)(x-3) > 0 \\ & \quad x \in (0, 2) \cup (3, \infty) \\ 0 & ; \ x(x-2)(x-3) = 0 \\ & \quad x = 0, 2, 3 \\ -1 & ; \ x(x-2)(x-3) < 0 \\ & \quad x \in (-\infty, 0) \cup (2, 3) \end{cases}$$




so, f(g(x)) is disc. at exactly points 0, 2 & 3

**Sol.5 C**

$$y = \frac{1}{t^2 + t - 2}, \quad t = \frac{1}{x-1}$$

$$y = \frac{1}{\frac{1}{(x-1)^2} + \frac{1}{x-1} - 2}$$

$$y = \frac{(x-1)^2}{1+(x-1)-2(x-1)^2}$$

$$y = \frac{x^2-2x+1}{x-2x^2-2+4x} = \frac{x^2-2x+1}{-2x^2+5x-2}$$

$$y = \frac{(x-1)^2}{-2x^2+4x+x-2} = \frac{(x-1)^2}{-2x(x-2)+1(x-2)}$$

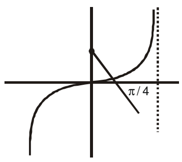
$$y = \frac{(x-1)^2}{(x-2)(-2x+1)}$$

by $\Rightarrow x \in R - \left\{2, +\frac{1}{2}\right\}$ so disc. at 1/2 & 2 let we also include x = 1 because at x = 1 ‘t’ is not defined.

**Sol.6 B**

$2 \tan x + 5x - 2 = 0$

$\tan x = -\frac{5x}{2} + 1$




**Sol.7 B**

$f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$

$$f(0^+) = \lim_{h\to 0} \frac{h\left(\sqrt{h} - \sqrt{h+1}\right)}{h}$$

$$= \lim_{h\to 0} \frac{h-h-1}{\sqrt{h}+\sqrt{h+1}} = -1$$

**Sol.8 B**

$$f(x) = \begin{cases} x\dfrac{(3e^{1/x}+4)}{2-e^{1/x}} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

$$f'(0^+) = \lim_{h\to 0} \frac{h\dfrac{(3e^{1/h}+4)}{2-e^{1/h}} - 0}{h}$$

$$= \lim_{h\to 0} \frac{3\left(1+\frac{1}{h}\right)+4}{2-1-\frac{1}{h}}$$

$$= \lim_{h\to 0} \frac{7h+3}{h-1} = 3$$

$$f'(0^-) = \lim_{h\to 0} \frac{-h\dfrac{(3e^{-1/h}+4)}{2-e^{-1/h}} - 0}{-h}$$

$$= \lim_{h\to 0} \frac{3\left(1-\frac{1}{h}\right)+4}{2-\left(1-\frac{1}{h}\right)}$$

$$= \frac{7h-3}{h+1} = -3$$

so, not diff. at x = 0

**Sol.9 B**

$$f(x) = \frac{x}{\sqrt{x+1}-\sqrt{x}} = \frac{x(\sqrt{x+1}+\sqrt{x})}{x+1-x}$$

$f(x) = x\left(\sqrt{x+1}+\sqrt{x}\right)$

Now, RHD

$$f(0^+) = \lim_{h\to 0} = \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h(\sqrt{h+1}-\sqrt{h})-0}{h}$$

$= 1$

since $ve^-$ values are not in domain of f(x) hance diff will lie diudid by RHD. Since RHD is finit hence f(x) is diff.

**Sol.10 B**

$f(x) = \sin^{-1}(\cos x)$

$$f(0^+) = \lim_{h\to 0} \frac{\sin^{-1}(\cos h) - \frac{\pi}{2}}{h}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}\sin\left(\frac{\pi}{2}-h\right) - \frac{\pi}{2}}{h}$$

$$f(0^+) = \lim_{h\to 0} \frac{\frac{\pi}{2} - h - \frac{\pi}{2}}{h} = -1$$

$$f'(0^-) = \lim_{h\to 0} \frac{\sin^{-1}(\cos(-h)) - \frac{\pi}{2}}{-h}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(\cos h) - \frac{\pi}{2}}{-h}$$

$$= \lim_{h \to 0} \frac{\frac{\pi}{2} - h - \frac{\pi}{2}}{-h} = 1$$

**Sol.11 B**

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x^2 + 1} & , \ 1 < x \le 2 \\ \dfrac{x^3 - x^2}{4} & , \ 2 < x \le 3 \\ \dfrac{9}{4}(|x - 4| + |2 - x| & , \ 3 < x < 4 \end{cases}$$

$$f'(2^+) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(2+h)^2\{2+h-1\}}{4} - \frac{3}{5}}{h}$$

$$= \lim_{h \to 0} \frac{5(h^3 + 4h^2 + 4h + h^2 + 4h + 4) - 12}{20h}$$

$$= \lim_{h \to 0} \frac{5(h^3 + 5h^2 + 8h).+8}{20h} = \text{Not exists}$$

Hence f(x) is not diff at x = 2

$$f'(3^+) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{9}{4}(|h-1| + |2-3-h|) - \frac{18}{4}}{h}$$

$$= \frac{9}{4} \lim_{h \to 0} \frac{-h+1+1+h-2}{h} = 0$$

$$f'(3^-) = \lim_{h \to 0} \frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h \to 0} \frac{\frac{(3-h)^3 - (3-h)^2}{4} - \frac{18}{4}}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} \frac{(3-h)^2(3-h-1) - 18}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} \frac{18 - 9h + 2h^2 - h^3 - 12h + 6h^2 - 18}{h}$$

$$= \frac{1}{4} \lim_{h \to 0} -9 - 12 = -\frac{21}{4}$$

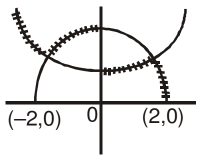
since $f'(3^+) \ne f'(3^-)$

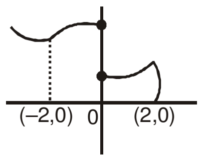
Hence f(x) is now diff at x = 3.

**Sol.12 D**

$$f(x) = \begin{cases} \max(\sqrt{4 - x^2}, \sqrt{1 + x^2}) & -2 \le x \le 0 \\ \min(\sqrt{4 - x^2} \ \sqrt{1 + x^2}) & 0 < x \le 2 \end{cases}$$

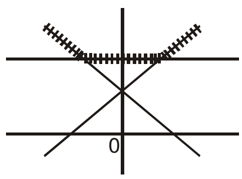
$y = 4 - x^2$ , $y = 1 + x^2$




**Sol.13 B**

f(x) = max {a – x, a + x, b}




so not diff. at two points

**Sol.14 B**

If f is differentiable everywhere.
then |f| will also be diff. everywhere.
and if two fns. are diff. then sum of then will also be diff. everywhere

**Sol.15 D**

f(x + y) = f(x) . f(y), f(3) = 3

f ′(0) = 11, f(3) = ?

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x) \cdot f(h) - f(x)}{h}$$

$$= f(x) \cdot \lim_{h \to 0} \frac{f(h) - 1}{(h)}$$

$$f'(3) = f(3) \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$f'(3) = f(3) \cdot f'(0)$

$f'(3) = 3 \times 11 = 33$

$[\because \ f(0) = f(0) \cdot f(0) \Rightarrow f(0) = 1]$

**Sol.16 D**

f(x + 2y) = f(x) + f(2y) + 2xy

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x)+f(h)-f(x)+2xy}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(0+h)-(0)}{h} + 2x$$

$$f'(x) = f'(0) + 2$$

**Sol.17 C**

$f(x) = x - x^2$

$$g(x) = \begin{cases} \max f(t), & 0 \le t \le x,\ 0 \le x \le 1 \\ \sin \pi x, & x > 1 \end{cases}$$

max f(t) will be obtained when t = x. so

max (f(t)) = $x - x^2$

$$\text{so}\quad f'(1^+) = \lim_{h\to 0} \frac{\sin \pi(1+h) - 0}{h}$$

$$= \lim_{h\to 0} -\frac{\sin \pi h}{\pi h}\pi = -\pi$$

$$f'(1^-) = \lim_{h\to 0} \frac{(1-h)^2-(1-h)^2-0}{-h}$$

Not diff. at x = 1 but cont.

**Sol.18 B**

$g(x) = x - [x]$ $f(0) = f(1)$

$h(x) = f(g(x))$

Let $x = a \in I$

$$h(a^+) = \lim_{x\to a^+} f(\{x\}) = f(0)$$

$$h(a^-) = \lim_{x\to a^-} f(g(x)) = f(1)$$

$h(a^+) = h(a^-)$ hence then is cont.

**Sol.19 D**

$$f(x) = \begin{cases} \log_{(4x-3)}(x^2-2x+5) & ; \ \frac{3}{4} < x < 1 \ \&\ x > 1 \\ 4 & x = 1 \end{cases}$$

$$\text{LHS } f(1^-) = \lim_{h\to 0} \log_{1-3h}\{(1-h)^2 - 2(1-h) + 5\}$$

$$= \lim_{h\to 0} \log_{(1-3h)}\{h^2 + 1 - 2h - 2 + 2h + 5\}$$

$$= \lim_{h\to 0} \log_{(1-3h)}(h^2+4)$$

$$= \lim_{h\to 0} \log(h^2+4) \times \frac{-3h}{\log(1-3h)} \times \frac{1}{-3h} = \infty$$

similarly $f(1^+)$ will be $\infty$.

**Sol.20 C**

$f(x) = x^2, \quad x \in Q^c$

$\quad\ = 1, \quad x \in Q$

By short trick

$x^2 = 1 \Rightarrow x = \pm 1$

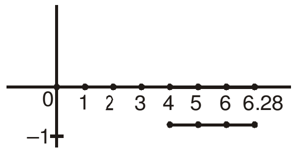
Hence f(x) will be const. at $x = \pm 1$

**Sol.21 C**

$f(x) = [\sin [x]]$

we'll define the given function as follows :-

$$[\sin [x]] = \begin{cases} 0 & ; \ 0 \le x < 1 \\ 0 & ; \ 1 \le x < 2 \\ 0 & ; \ 2 \le x < 3 \\ 0 & ; \ 3 \le x < 4 \\ -1 & ; \ 4 \le x < 5 \\ -1 & ; \ 5 \le x < 6 \end{cases}$$




so point where function is not cont. is (4, –1)

**Sol.22 B**

$$f(x) = \lim_{t\to\infty} \left\{\frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}\right\}$$

$$\lim_{t\to 0} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1} \begin{cases} 0 & 1+\sin \pi x = 1 \\ & \sin \pi x = 0 \\ & \pi x = n\pi \\ -1 & x = n, n = 0,1,2,...... \\ & 1+\sin \pi x > 1 \\ & \sin \pi x > \sin 0 \\ & x > n \\ -1 & 1 > 1+\sin \pi x > 0 \\ & \pi x > -\frac{\pi}{2} \\ & 0 > x > -\frac{n}{2} \end{cases}$$

$$\text{Now } f(x) = \begin{cases} 0 & x = n, n = 1,2,3... \\ -1 & x > n \\ -1 & -\frac{n}{2} < n < 0 \end{cases}$$

| | | | |
|---|---|---|---|
| (i) | $f(0^+) = -1$ | (ii) | $f(1^+) = -1$ |
| | $f(0^+) = -1$ | | $f(1^-) = -1$ |
| | $f(0) = 0$ | | $f(1) = 0$ |

Similarly for all integer the function will the disc.

**Sol.23 B**

$$f(x) = \begin{cases} \sqrt{x}\left(1+\sin\frac{1}{x}\right), & x > 0 \\ -\sqrt{x}\left(1+\sin\frac{1}{x}\right), & x < 0 \\ 0, & x = 0 \end{cases}$$

$$f'(0^+) = \frac{f(0+h)-f(0)}{h} \Rightarrow \lim_{h\to 0} d$$

$$= \lim_{h\to 0} \frac{\left(1+\sin\frac{1}{h}\right)}{\sqrt{h}} = \text{m.d.} \Rightarrow \text{N. diff.}$$

$$f(0^+) = \lim_{h\to 0} \sqrt{h}\left(1+\sin\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{h}{\sqrt{h}}\left(1+\sin\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{1}{\sqrt{h}}\right)\frac{\left(1+\sin\frac{1}{h}\right)}{(1/h)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{h}}\left\{h + h\sin\frac{1}{h}\right\}$$

$= 0$

$$f(0^-) = \lim_{h\to 0} -\sqrt{h}\left(1-\sin\frac{1}{h}\right)$$

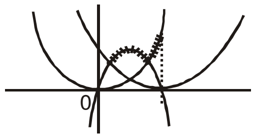
$= 0$

Hence (B)

**Sol.24 C**

$f(x) = \max\{x^2, (x-1)^2, 2x(1-x)\}$

$0 \le x \le 1$




so, (c)

**Sol.25 D**

$[n + p \sin x] = n\, [p \sin x]$

$\therefore$ $[p \sin x]$ is non. diff. where $p \sin x$ is as integer but P is prime and $0 < \sin x \le 1$ $[0 < x < \pi]$

$\therefore$ $p \sin x$ is an integer only when

$\sin x = \frac{r}{p}$ ; where $0 < r \le p$ and $r \in N$

For $r = p$ ; $\sin x = 1 \Rightarrow x = \frac{\pi}{2}$ in $(0, \pi)$

For $0 < r < p$ ; $\sin x = \frac{r}{p}$

$x = \sin^{-1}\left(\frac{r}{p}\right)$ or $\pi - \sin^{-1}\left(\frac{r}{p}\right)$

Number of such values of $x = P - 1 + P - 1 = 2P - 2$

Total No. of points $= 2P - 2 + 1 = 2P - 1$

**Sol.26 C**

$f(x) = x^3 - x^2 + x + 1$

$$g(x) = \begin{cases} \max(f(t)); 0 \le t \le x & \text{for } 0 \le x \le 1 \\ x^2 - x + 3; 1 < x \le 2 \end{cases}$$

max {f(t)} will be obtained when 't' would be max.

so, t = x.

so, $\max\{f(t)\} = x^3 - x^2 + x + 1$

$$f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2-(1+h)+3-2}{h}$$

= not defined

so not derivable

Now check cont by,

$$f(1^+) = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} (1+h)^2 - (1+h) + 3$$

$= 3$

& f(1) = 2 so f(x) is not cont.

**Sol.27 C**

By using L' Hospital rule

$$= \lim_{x\to 0} \frac{2f'(x) - f'(2x) + 4f'(4x)}{2x}$$

Again

$$= \lim_{x\to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2} = 12$$

**Sol.28 C**

Put $y = 0 \Rightarrow f\left(\frac{x}{3}\right) = \frac{f(x)}{3} \Rightarrow f(3x) = 3f(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{f(3x)+f(3h)}{3}-f(x)}{h}=\lim_{h\to 0}\frac{\frac{8f(x)+3f(x)}{3}-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=f'(0)$$

$f'(x) = 3 \Rightarrow f(x) = 3x + c \Rightarrow f(x) = 3x$

**Sol.29 C**

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)+f(h)+xh-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)}{h}+x$$

$f'(x) = x + 3$

$f(x) = \frac{x^2}{2} + 3x + c \qquad f(0) = 0$

$f(x) = \frac{x^2}{2} + 3x \qquad \Rightarrow c = 0$

**Sol.30 D**

Put $x = 0$, $y = 0 \Rightarrow f(0) = \frac{4}{7}$

Now put $y = 0$

$$f\left(\frac{x}{3}\right)=\frac{4-2[f(x)+f(0)]}{3}$$

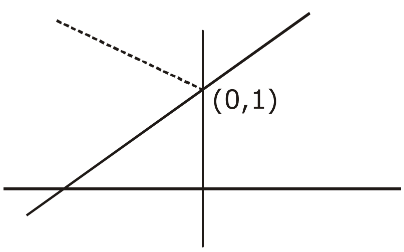
$\Rightarrow 3f(x) = 4 - 2\,[f(3x) + 6(0)]$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Now proceed as in question (28)

$f(x) = \frac{4}{7}$

**Sol.31 C**




**Sol.32 D**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{1}{x}-\frac{2}{e^{2x}-1}\right)$$

$$=\lim_{x\to 0}\left(\frac{e^{2x}-1-2x}{x(3^{2x}-1)}\right)$$

$$=\lim_{x\to 0}\frac{e^{2x}-1-2x}{2x^2}=1$$

**Sol.33 C**

$$\frac{|f(x)-f(y)|}{x-y}\le (x-y)$$

$$\lim_{x\to y}\frac{|f(x)-f(y)|}{x-y}\le \lim_{x\to y}(x-y)$$

$f'(x) = 0 \Rightarrow f(x) = C \Rightarrow f(0) = 0 \Rightarrow C = 0$
$f(1) = 0$

**Sol.34 D**

$f(x) = |x - 1| + |x - 2| + \cos x$
All three fns are cont. is [0, 4]
so sum of all these functions is also
a cont. funs.

**Sol.35 D**

$g\left(\frac{1}{2}\right) = f(1) = 0$

$f\left(\frac{1^+}{2}\right) = f[1^+] = f(1) = 0$

$g\left(\frac{1^-}{2}\right) = f[0] = f(0) = 1$

Discont. at $x = \frac{1}{2}$

**Sol.36 B**

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x).f(h)-f(x)}{h}=f(x)\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$f'(x) = f(x)\,.\,f'(0)$
$f'(x) = 2f(x)$
$\ln f(x) = 2x + C \quad : C = 0$
$f(x) = p^{2x}$

**Sol.37 B**

$$f'(x) = \lim_{h\to 0} \frac{f(x+n) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x).f(h) - f(x)}{h} = f(x) \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

$f'(x) = f(x) \,.\, f'(0)$
$f'(x) = 2f(x)$
$\ln f(x) = 2x + C \quad : C = 0$
$f(x) = e^{2x}$

**Sol.38 C**

By doing rationlize

$$f(0) = \lim_{x\to 0} \frac{(a^2 - ax + x^2) - (a^2 + ax + x^2)}{(x+x) - (a-x)}$$

$$\times \frac{(\sqrt{a+x} + \sqrt{a-x})}{(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2})}$$

$$= -\frac{-2ax}{2x}\left(\frac{2\sqrt{a}}{2a}\right)$$

$f(0) = -\sqrt{a}$

**Sol.39 C**

$$\text{RHL} = \lim_{x\to \frac{\pi^+}{2}} f(x) = \lim_{h\to 0} \frac{\sin\left\{\cos\left(\frac{\pi}{2} + h\right)\right\}}{h}$$

$$= \lim_{h\to 0} \frac{\sin\{-\sin b\}}{h} = \lim_{h\to 0} \frac{\sin(1 - \sin h)}{h} \to \infty$$

DNE

**Sol.40 B**

$$f(x) = \left[\sqrt{2}\sin\left(\frac{\pi}{4} + h\right)\right]$$

$-\frac{1}{\sqrt{2}} \Downarrow (2) \qquad O \Downarrow (1) \qquad \frac{1}{\sqrt{2}} \Downarrow (2)$

Total solutions = 5

**Sol.41 D**

$$f(1^+) = \lim_{x\to 1^+} x^2 \left[\frac{1}{x^2}\right] = 0$$

$$f(1^-) = \lim_{x\to 1^-} x^2 \left[\frac{1}{x^2}\right] = 1$$

Discont. at $x = 1$
similarly for $x = -1$

$$f(x) = x^2 \left(\frac{1}{x^2} - \left\{\frac{1}{x^2}\right\}\right) = 1 - x^2 \left\{\frac{1}{x^2}\right\}$$

$$f(0^+) = \lim_{x\to O^+} 1 - x^2 \left\{\frac{1}{x^2}\right\} = 1$$

$f(0^{-1}) = 1 \qquad$ But $f(0) = 0$
So dicont. at $x = 0$
at $x = 2$, RHL = LHL = $f(2) = 0$
coint. at $x = 2$

**Sol.42 D**

$$\text{RHL} = \lim_{h\to 0} \sin[\ell nh] = [-1, 1]$$

$$\text{LHL} = \lim_{h\to 0} \sin[\ell n\, h] = [-1, 1]$$

So DNE

**Sol.43 D**

$$f(x) = \frac{|x-3|}{|x-2|} + \frac{1}{1+[x]}$$

$x \neq 2 \qquad 1 + [x] = 0$
$[x] \neq -1, \qquad x \in [1, 0)$
And $[x]$ will be disoint. at every integer
So $x \in R - \{(-1, 0) \cup n, n \in I\}$

**Sol.44 B**

$f(x)$ should be a constant function.

**Sol.45 C**

$$\text{RHL} = \lim_{h\to 0} \frac{a^h - 1}{h} = \ell na$$

$$\text{LHL} = \lim_{h\to 0} \frac{a^{-1-h} - 1}{-1-h} = 1 - \frac{1}{a}$$

$f(x) = \ell na \quad \Rightarrow$ Discont. at $x = 0$

**Sol.46 D**

$f'(O^+) = p + q \qquad ....(1)$
$f'(O^-) = -p + q \qquad ....(2)$
$f'(O^+) = f'(O^-) \;\Rightarrow\; p + q = 0, r \in R$

**Sol.47 A**

$g(x) = [x] + 1$

$h(x) = g(\sin x) = [\sin x] + 1$

$[\sin x]$ is discont at $x = \frac{\pi}{2}$

$\Rightarrow$ $[\sin x] + 1$ is also a discont at $x = \frac{\pi}{2}$

**Sol.48 B**

$f(x) = [\tan^2 x]$

$\text{RHL} = \lim_{x \to 0^+} [\tan^2 x] = 0$

$\text{LHL} = \lim_{x \to 0^-} [\tan^2 x] = 0 \quad : f(x) = 0$

So cont. at $x = 0$

**Sol.49 C**

$f(x) = [x]^2 + \sqrt{(x-[x])^2}$

DIscont. at every integer because $[x]$ is discont. at every integer.

But $f(x)$ is cont. at $x = 1$

So option (C) is correct.

**Sol.50 B**

$f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \ell \neq 0$

$f'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = -\ell \neq 0$

Non. diff. But cont. at $x = 0$

**Sol.51 B**

$f(x) = \frac{3 - 2\sqrt{3} + 2x - x^2}{x - \sqrt{3}}$

$= \frac{(\sqrt{3} - x)(\sqrt{3} + x) + 2(x - \sqrt{3})}{x - \sqrt{3}}$

$f(x) = 2 - (\sqrt{3} + x)$

$f(\sqrt{3}) = 2 - 2\sqrt{3} = 2(1 - \sqrt{3})$

**Sol.52 C**

$f(x) = \text{Sgn}(4 - 2\sin^2 x - 2\sin x)$

$= \text{Sgn}[(\sin x + 2)(2 - 2\sin x)]$

$f(x) = 0 \qquad$ when $x > \frac{\pi}{2}$

$= 1 \qquad x < \frac{\pi}{2}$

$= -1 \qquad \sin x > 1$ not possible

SO isolated pt. discont.

**Sol.53 A**

$\text{RHL} \Rightarrow x = 0 + h$

$\lim_{h \to 0} |g(f(h))|$

as $h \to 0$ $f(h) \to$ ; $g(0) \to 0$

$\text{RHL} = 0$

$h(0) = 0$

So cont. at $x = 0$

**Sol.54 B**

$f(1) = 0$

$$f(x) = \begin{cases} 1 & x > 1 \\ 0 & x < 1 \end{cases}$$

Discont. at $x = 1$

**Sol.55 A**

$$\text{RHL} = \lim_{x \to 0^+} \frac{[\{|x|\}]e^{x^2}\{[x + \{x\}]\}}{(e^{1/x^2} - 1)\text{Sgn}(\sin x)}$$

fraction part of greatest integer in always zero.

So RHL = LHL = 0

So coint. at $x = 0$

**Sol.56 D**

$$\text{RHL} = \lim_{x \to 0^+} \frac{\ln(e^{x^2} + 2\sqrt{x} + 1 - 1)(e^{x^2} + 2\sqrt{x} - 1)}{(e^{x^2} + 2\sqrt{x} - 1)\sqrt{x}}$$

$= 2$

$\text{LHL} = \lim_{x \to 0^-} \frac{x[x]^2 \ell n2}{\ell n(x+1)} = \ell n2$

Non-Removable disont at $x = 0$

**Sol.57 A**

$\text{RHL} = \lim_{x \to 0^+} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = 1$(Rationlize)

$\text{LHL} = \frac{1}{\sqrt{2}} f(g(x))$

$= \lim_{x \to 0^-} \frac{1}{\sqrt{2}} \frac{|\sqrt{2}\cos x| - |\sqrt{2}\sin x|}{\cos 2x}$

$$= \lim_{x \to 0^-} \frac{1}{\cos x - \sin x} = 1$$

cont. at x = 0

**Sol.58 D**

$$\lim_{x \to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - ....\right) - x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ....\right)}{x^2}$$

= 0

f(x) is cont at x = 0

**Sol.59 C**

$$\text{RHD} = \lim_{h \to 0} \frac{\frac{h}{\sinh} - 1}{h} = \lim_{h \to 0} \frac{h - \sinh}{h^2} = 0$$

$$\text{LHD} = \lim_{h \to O} \frac{\frac{-h}{\sinh} - 1}{h} = \lim_{h \to 0} \frac{-h - \sinh}{h^2} \to \infty$$

Non. diff. at x = 0

RHL = 1 Discont.

LHL = –1

**Sol.60 B**

$$f(x) = \begin{cases} x.\dfrac{a^{-2|x|} - 5}{3 + a^{1/|x|}} & ; |x| \neq 0; a > 1 \\ 0 & ; x = 0 \end{cases}$$

$f'(0^+) = 0$; $f'(0^-) = 0$

diff. & cont. at x = 0

**Sol.61 A**

LHD (x = 1) = RHD (x = 1)

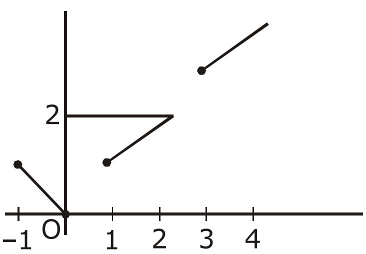
1 = 2a + b ....(1)

LHL (x = 1) RHL (x = 1)

1 = a + b + c ....(2)

b = 1-2a, c = a

**Sol.62 D**




Discont at x = 1, 2, 3

Non. diff. at x = 1, 2, 3

**Sol.63 D**

RHD (at x = 0) = 0; LHD = 1

RHD (at x = 1) = 2; LHD = 2

RHL (at x = 0) = 0 = LHL

RHL (at x = 1) = LHL (x = 1)

Diff. and cot. at x = 1

Non diff. x = 0 but cont. at x = 0

**Sol.64 D**

$$\text{RHD} = \lim_{h \to 0} \frac{h + h + h\sinh - 0}{h} = \lim_{h \to 0} 2 + \sin h = 2$$

$$\text{LHD} = \lim_{h \to 0} \frac{-h + (1 - h) - h\sin(1 - h)}{-h} \to \infty$$

Non. diff. at x = 0

$$\text{RHL} = \lim_{h \to 0} h + h + h \sin h = 0$$

$$\text{LHL} = \lim_{h \to 0} -h + 1 - h + h \sin(1 - h) = h$$

discont at n = 0

## EXERCISE – II HINTS & SOLUTIONS

**Sol.1** **A,B,C**

**(A)** $f(x) = \dfrac{1}{1+2^{1/x}}$

RHL = 0

LHL = 1

**(B)** Not defined at x = 0

RHL $= -\dfrac{\pi}{2}$

LHL $= \dfrac{\pi}{2}$

**(C)** $f(x) = \dfrac{e^{1/x}-1}{e^{1/x}+1}$

RHL = 1 ; LHL = –1

**(D)** $f(x) = \dfrac{1}{\ell n\,|x|}$

**Sol.2** **B,C,D**

(B) $x = 1 - x \Rightarrow x = 1/2$

(C) $x = 0$

(D) $x = -x \Rightarrow x = 0$

**Sol.3** **A,B,C**

$\lim\limits_{x\to 1^+} |x-3| = 2$

$\lim\limits_{x\to 1^-} \left(\dfrac{x^2}{4}\right) - \left(\dfrac{3x}{2}\right) + \dfrac{13}{4} = 2$

function is cont. at x = 1

function is also diff. at x = 1

and will be cont. at x = 3

**Sol.4** **A,B,D**

tan x will be discontinuous at $x = \dfrac{\pi}{2}$

and $|x-0.5|$ and $|x-1|$ will be non-differentiable at x = 0.5 and x = 1 respectively.

so non diff. at $x = \dfrac{1}{2}, 1, \dfrac{\pi}{2}$

**Sol.5** **A,B,C**

$f(x) = [x]$ ; $g(x) = \begin{cases} 0 & , x \in I \\ x^2 & , x \in R - I \end{cases}$

for g(x)

RHL (at x = 1) = 1 g(1) = 0

LHL (at x = 1) = 1

so discont. at x = 1 (A)

for f(x)

RHL (at x = 1) = 1

LHL (at x = 1) = 0 dis cont. at x = 1 (B)

For gof(x)

$gof(x) = [x]^2$ , $\lim\limits_{x\to 1} [1]^2 = 1$ cont. $\forall\ x \in R$

**Sol.6** **B,D**

$f(x) = \begin{cases} 3 - \left[\cot^{-1}\left(\dfrac{2x^3-3}{x^2}\right)\right] & x > 0 \\ \{x^2\}\cos(e^{1/x}) \quad \text{for} & x < 0 \end{cases}$

RHL $\lim\limits_{x\to 0^+} 3 - \left[\cot^{-1}\dfrac{2x^2-3}{x^2}\right] = 3 - 3 = 0$

$\cot^{-1}(-\infty) \to [\pi] = 3$

LHL $\lim\limits_{x\to 0^-} \{x^2\} \cos e^{1/x}$

$x = 0 - h$

$\lim\limits_{h\to 0} \underset{0}{\underset{\downarrow}{\{h^2\}}} \times \underset{1}{\underset{\downarrow}{\underbrace{\cos e^{-1/h}}}} = 0$

**Sol.7** **A,B,C**

$f(x) = [x] + \sqrt{x-[x]}$

$f(x) = [x] + \sqrt{\{x\}} \Rightarrow x - \{x\} + |\{x\}|$

$f(x) = x$

f(x) is cont. on R, $R^+$, R – I

**Sol.8** **A,C**

$f(x) = \sum\limits_{k=0}^{n} a_k\,|x|^k$

$f(x) = a_0|x|^0 + a_1|x| + a_2|x|^2 + .......+ a_n|x|^n = f(|x|)$

f(x) is cont. at x = 0 $\forall$ all is

2k + 1 means all odd $a_i$'s

$f(x) = a_0 + a_2x^2 + a_4x^4 + .........$

f(x) will be diff. at x = 0

**Sol.9** **A,B,D**

f(0) = 0

$f(0^+) = [0^+] = 0$

$f(0^-) = [0^+] = 0$

$f(0^-) = [0^+] = 0$

So f(x) is cont. at x = 0

f(1) =0

$f(1^+) = -1$ So discont. at x = 1

$\Rightarrow$ Non. diff. at x = 1

**Sol.10 B,D**

$f(x) = \lim_{n\to\infty} (\sin x)^{2n}$

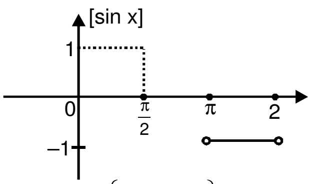
$$f(x) = \begin{cases} 1 \ ; & x = \pi/2 \\ 0 \ ; & x < \pi/2 \\ \infty \ ; & x > \pi/2 \end{cases}$$

$$\left.\begin{matrix} f\left(\frac{\pi}{2}^{+}\right) = \infty \\ f\left(\frac{\pi}{2}^{-}\right) = 0 \end{matrix}\right\} \text{function is not cont. at } x = \frac{\pi}{2}$$

function is discont at x = $\frac{\pi}{2}$ & infinit number of points.

**Sol.11 A,B,D**

$f(x) = \frac{1}{[\sin x]}$




$D_f$ : [sin x] $\neq$ 0

$x \in (2n\pi + \pi, 2n\pi + 2\pi) \cup \left\{2n\pi + \frac{\pi}{2}\right\}$

cont. when $x \in (2n\pi + \pi, 2n\pi + 2\pi)$

f(x) has the period of $2\pi$

**Sol.12 A,B,D**

$f(x) = \sqrt{1-\sqrt{1-x^2}}$

$D_f$ : $1 - x^2 \geq 0 \Rightarrow -1 \leq x \leq 1$

RHL (at x = 0) = 0

LHL (at x = 0) = 0 cont. at x = 0

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1-\sqrt{1-h^2}} - 0}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \sqrt{1-\sqrt{1-h^2} \times \frac{1+\sqrt{1-h^2}}{1+\sqrt{1+h^2}}}$$

$$= \lim_{h\to 0} \frac{1}{h} \sqrt{\frac{1-1+h^2}{1+\sqrt{1+h^2}}} = \frac{1}{2}$$

$$\text{LHD} = -\frac{1}{2}$$

**Sol.13 A,C**

$$f(x) = \lim_{n\to\infty} \frac{1-x^n}{1+x^n} = \begin{cases} 1 \ ; & x < 1 \\ \infty \ ; & x > 1 \\ 0 \ ; & x = 1 \end{cases}$$

$f(1^+) = \infty$

$f(1^-) = 1$

f(x) is a constant in 0 < x < 1

$f'(0^+) \neq f'(0^-)$ not diff. at x = 1

**Sol.14 B,C**

$$f\left(\frac{1}{4^n}\right) = (\sin e^n)\, e^{-n^2} + \frac{n^2}{1+n^2}$$

put n = $\infty$

f(0) = [{a finite quantity b/w (–1, 1)} × 0] + 1 = 1

**Sol.15 C,D**

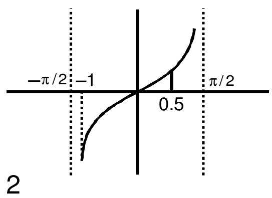
$f(x) = \frac{x}{2} - 1$ on $[0, \pi]$

$f(x) = \frac{x-2}{2}$ : $\frac{1}{f(x)} = \frac{2}{x-2}$ $\quad 0 \leq \frac{x}{2} \leq \frac{\pi}{2}$

$f^{-1}(x) = 2(1 + x)$ is cont. $\quad -1 \leq \frac{x}{2} - 1 < \frac{\pi}{2} - 1 \approx 0.5$

$\tan f(x) = \tan\left(\frac{x-2}{2}\right)$

is cont.




$\frac{1}{f(x)}$ will discont at x = 2

**Sol.16 A,C**

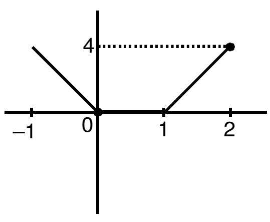
f(x) = |[x]x| $\quad -1 \leq x \leq 2$

$$f(x) = \begin{cases} -x & -1 \leq x < 0 \\ 0 & 0 \leq x < 1 \\ x & 1 \leq x < 2 \\ 4 & x = 2 \end{cases}$$

$\lim_{x\to 0^+} 0 = 0 : \lim_{x\to 0^-} 0 = 0$

cont. at x = 0

Not diff. at x = 2



**Sol.17 A,B**

$f(x) = 1 + x\,.\,[\cos x] \qquad 0 < x \le \dfrac{\pi}{2}$

$f\left(\dfrac{\pi}{2}\right) = 1 = f\left(\dfrac{\pi^-}{2}\right)$

function is cont. is $\left(0, \dfrac{\pi}{2}\right]$

$f'\left(\dfrac{\pi^-}{2}\right) = \lim\limits_{h\to 0} \dfrac{1 - h[\cos(-h)] - 1}{-h} = 1$

diff. at $x = \dfrac{\pi}{2}$

**Sol.18 B,D**

$f(x) = (\sin^{-1} x)^2 \,.\, \cos\left(\dfrac{1}{x}\right)$ if $x \ne 0$

$= 0$ if $x = 0$

LHL = RHL = $\lim\limits_{x\to 0} (\sin^{-1} x)^2 \cos\left(\dfrac{1}{x}\right)$

= 0 × [a finite quantity b/w (–1, 1)]

= 0

$f'(0^+) = \lim\limits_{h\to 0} \dfrac{(\sin^{-1} h)^2 \cos\left(\dfrac{1}{h}\right)}{h}$

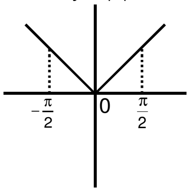
$= \lim\limits_{h\to 0} \left(\dfrac{\sin^{-1} h}{h}\right) (\sin^{-1} h) \cos(1/h)$

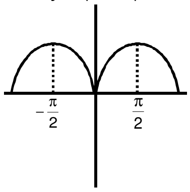
= 1 × (0) × (a finite quantity)

= 0

$f'(0^-) = 0$

**Sol.19 B,D**




Not diff. at x = 0

**Sol.20 A,B,C**

$f(x) = 3(2x + 3)^{2/3} + 2x + 3$

$f\left(\dfrac{-3}{2}\right) = 0 - 3 + 3 = 0$

cont. every where

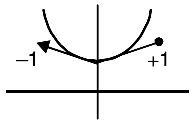
$f'(x) = -2\,(2x + 3)^{-1/3} + 2$

$= -\dfrac{2}{(2x+3)^{1/3}} + 2$

at $x = -\dfrac{3}{2}$ ; $f'(x)$ is not defined

**Sol.21 B,D**

$f(x) = 2 + |\sin^{-1} x|$




function is continuous everywhere in its domain but f(x) is not diff. at x = 0

**Sol.22 A,B,D**

$f(x) = x^2 \sin\left(\dfrac{1}{x}\right) \quad , \quad x \ne 0$

$= 0 \quad , \quad x = 0$

cont. at x = 0

$f(0^+) = f(0^-) = f(0) = 0$

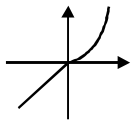
$f'(0^+) = \lim\limits_{h\to 0} \dfrac{h^2 \sin 1/h}{h} = \lim\limits_{h\to 0} h \sin \dfrac{1}{h} = 0$

$f'(0^-) = 0$

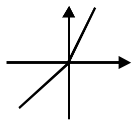
Diff. at x = 0

**Sol.23 A,B,D**

(A)

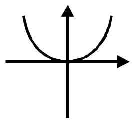


(B)

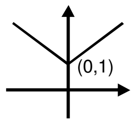


(C) $h(x) = x^2 \quad x \ge 0$

$= -x^2 \quad x < 0$



(D)




**Sol.24 A,B,D**

$\sin^{-1} x + |y| = 2y$

$\sin^{-1} x = 2y - y$

$y = \sin^{-1} x$

y is defined for $-1 \le x \le 1$

## EXERCISE – III HINTS & SOLUTIONS

**Sol.1** $f(x) = \dfrac{3x^3 + ax + a + 3}{x^2 + x - 2}$

$\lim\limits_{x \to -2} \dfrac{3x^2 + ax + a + 3}{x^2 + x - 2} = f(-2)$

$\Rightarrow$ format is $\dfrac{15-a}{0}$, for existence of limit $N^r = 0$

$\Rightarrow \quad a = 15$

$\lim\limits_{x \to -2} \dfrac{3x^2 + 15x + 18}{x^2 + x - 2} = f(-x)$

R.H.S. $\lim\limits_{h \to 0} \dfrac{3(-2+h)^2 + 15(-2+h)}{(-2+h)^2 + (-2+h) - 2} = f(-2)$

$\lim\limits_{h \to 0} \dfrac{12 + 3h^2 - 12h - 30 + 15h + 18}{h^2 - 3h} = f(-2)$

$\lim\limits_{h \to 0} \dfrac{3h^2 + 3h}{h^2 - 3h} = f(-2)$

$\boxed{f(-2) = -1}$

**Sol.2** $f(x) = \begin{cases} |ax + 3| & x \le -1 \\ |3x + a| & -1 < x \le 0 \\ \dfrac{b \sin 2x}{x} - 2b & a < x < \pi \\ \cos^2 x - 3 & x \ge \pi \end{cases}$

$f(0^-) = f(0)$

$f(x^-) = \dfrac{-b \sin 2h}{-h} - 2b = |a|$

$+ 2b - 2b = |a| \quad \Rightarrow \quad x = 0$

$f(\pi^-) = f(\pi)$

$\lim\limits_{h \to 0} \dfrac{b \sin(\pi - h)}{(\pi - h)} - 2b = -2$

$-2b = -2 \qquad \Rightarrow \qquad b = 1$

So, $\boxed{a = 0}$ & $\boxed{b = 1}$

**Sol.3** $f(x) = \begin{cases} \dfrac{\ell n \cos x}{\sqrt[4]{1 + x^2} - 1} & x > 0 \\ \dfrac{e^{\sin 4x} - 1}{\ell n(1 + \tan 2x)} & x < 0 \end{cases}$

$f(0^+) = \lim\limits_{h \to 0} \dfrac{\ln \cosh}{(1 + h^2)^{1/4} - 1}$

$= \lim\limits_{h \to 0} \dfrac{\ln(1 + \cosh - 1)\}(\cosh - 1)}{(\cosh - 1)\{1 + h^2)^{1/4} - 1\}}$

$= \lim\limits_{h \to 0} \dfrac{(\cos h - 1)}{(1 + h^2)^{1/4} - 1} \Rightarrow \lim\limits_{h \to 0} \dfrac{\dfrac{\cosh - 1}{h^2} \times h^2}{\left(x + \dfrac{h^2}{4} - 1\right)}$

$\Rightarrow \qquad -\dfrac{1}{2} \times 4 = -2$

$f(0^-) = \lim\limits_{h \to 0} \dfrac{e^{\sin(-4h)}}{\ell n(1 - \tan 2h)}$

$= \lim\limits_{h \to 0} \dfrac{\{e^{-\sin 4h} - 1\}(-\tan 2h)}{\ell n\{1 + (-\tan 2h)\}(-\tan 2h)} = \lim\limits_{h \to 0} \dfrac{e^{-\sin 4h} - 1}{-\dfrac{\tan 2h}{2h} \times 2h}$

$= -\lim\limits_{h \to 0} \dfrac{e^{-\sin 4h} - 1}{2h} = \lim\limits_{h \to 0} \dfrac{e^{\sin 4h} - 1}{2h(e^{\sin 4h})}$

$= \lim\limits_{h \to 0} \dfrac{(e^{\sin 4h} - 1)\sin 4h}{\sin 4h \times 2h e^{\sin 4h}} = \lim\limits_{h \to 0} \dfrac{2}{e^{\sin 4h}} = 2$

$f(0) \to = -2$ & $f(0^+) = e$

Hence it is discontinuity of non removable type.

**Sol.4** $f(x) = x^3 - 3x^3 - 4x + 12$ &

$h(x) = \begin{cases} \dfrac{f(x)}{x - 3} & , x \ne 3 \\ k & , x = 3 \end{cases}$

$h(x) = \begin{cases} \dfrac{x^3 - 3x^2 - 4x + 12}{x - 3} & , x \ne 3 \\ k & , x = 3 \end{cases}$

$f(x) = f(3^+) = f(3)$

$\lim_{x\to 3} f(x) = f(3)$

$$\lim_{x\to 3} \frac{a^3 - 3x^2 - 4x + 12}{x - 3} = k$$

$\boxed{5 = k}$

Zero of f(x) $\Rightarrow$ x = 3, 2, –2

$$h(x) = \begin{cases} \dfrac{x^3 - 3x^2 - 4x + 12}{x - 3} & , x \neq 3 \\ k & , x = 3 \end{cases}$$

$h(x) = x^2 - 4$

& checking at x = –3

h(x) = 5 = k

hence then.

**Sol.5**

$$y_n(x) = x^2 + \frac{x^2}{1 + x^2} + \frac{x^2}{(1 + x^2)^2} + .... + \frac{x^2}{(1 + x^2)^{n-1}}$$

$$y_n(x) = \frac{x^2(1 + x^2)\left(1 - \dfrac{1}{(1 + x^2)^2}\right)}{x^2}$$

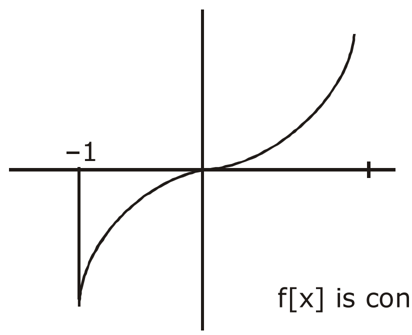
$$\operatorname{Lim}_{x\to 0} y_n(x) = \operatorname{Lim}_{x\to 0} (1 + x^2)\left[1 - \frac{1}{(1 + x^2)^n}\right] = 0 = y_n(0)$$

**Sol.6** $f(x) = x - |x - x^2|, x \in [-1, 1]$

$f(x) = x - |x - x^2| \qquad -1 \le x \le 1$

$f(x) = x(2 - x) \qquad -1 \le x < 0$

$\quad = x^2 \qquad 0 \le x \le 1$




f[x] is con is [–1, ]

f(x) is cont is [–1, 1]

**Sol.7**
$$f(x) = \begin{cases} \dfrac{1 - \sin \pi x}{1 + \cos 2\pi x} & x < \dfrac{1}{2} \\ P & x = \dfrac{1}{2} \\ \dfrac{\sqrt{2x - 1}}{\sqrt{4 + \sqrt{x - 1}} - 2} & x > \dfrac{1}{2} \end{cases}$$

$$f\left(\frac{1^-}{2}\right) = \lim_{h\to 0} \frac{1 - \sin \pi\left(\frac{1}{2} - h\right)}{1 + \cos 2\pi\left(\frac{1}{2} - h\right)}$$

$$= \lim_{h\to 0} \frac{1 - \cos \pi h}{1 - \cos 2\pi h} \Rightarrow \lim_{h\to 0} \frac{2\sin^2\left(\frac{\pi h}{2}\right)}{2\sin^2 \pi h}$$

$$f\left(\frac{1^-}{2}\right) = \lim_{h\to 0} \frac{\sin^2\left(\frac{\pi h}{2}\right)}{\frac{\sin^2 \pi h}{\pi^2 b^2} \times \pi^2 b^2} \times \frac{\pi^2 h^2}{4} \times \frac{4}{\pi^2 h^2}$$

$$= \frac{1}{4}$$

$$f\left(\frac{1^+}{2}\right) = \lim_{h\to 0} \frac{\sqrt{2\left(\frac{1}{2} + h\right) - 1}}{\sqrt{4 + \sqrt{2\left(\frac{1}{2} + h\right) - 1}} - 2}$$

$$= \lim_{h\to 0} \frac{\sqrt{2h}}{\sqrt{4\sqrt{2h} - 2}}$$

$$= \lim_{h\to 0} \frac{\sqrt{2h}}{\sqrt{2 + \sqrt{2h}}} = 0$$

There is no value of 'P' because f(x) is not cointinuous.

**Sol.8** $g(x) = \sqrt{6 - 2x}$

$h(x) = 2x^2 - 3x + a$

**(a)** $h(g(x)) \Rightarrow h(\sqrt{6 - 2\times 2}) \Rightarrow h\ \sqrt{2}$

$\Rightarrow 2 \times 2 - 3\sqrt{2} + a$

$\Rightarrow 4 - 3\sqrt{2} + a$

**(b)** $f(x) = \begin{cases} g(x) & ; \quad X \le 1 \\ h(x) & ; \quad x > 1 \end{cases}$

$$f(x) = \begin{cases} \sqrt{6-2x} & ; \quad X \le 1 \\ 2x^2 - 3x + a & ; \quad x > 1 \end{cases}$$

$$f(I^{-1}) = \lim_{h\to 0} \sqrt{6-2+2h} = h$$

$$f(I^{+}) = \lim_{h\to 0} 2(1+h^{-})^2 - (1+h) + 2$$

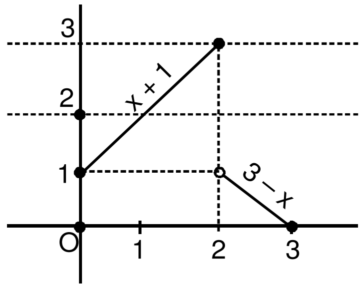
$$= -1 + a$$

If f(x) is cont. a – 1 = 2

$$\boxed{a = 3}$$

**Sol.9** $f(x) = \begin{cases} 1+x & ; \quad 0 \le x \le 2 \\ 3-x & ; \quad 2 < x \le 3 \end{cases}$

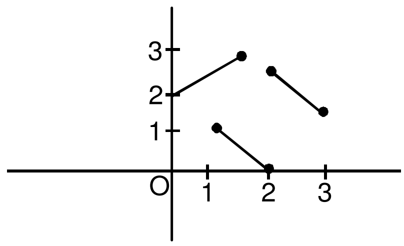
$$g(x) = f(f(x)) = \begin{cases} 1+f(x) & ; \quad 0 \le f(x) \le 2 \\ 3-f(x) & ; \quad 2 < f(x) \le 3 \end{cases}$$




$$0 \le f(x) \le 2 \begin{cases} \longrightarrow 1 + x + 1 \; ; \; 0 \le x \le 1 \\ \longrightarrow 1 + 3 - x \; ; \; 2 < x \le 3 \end{cases}$$

$$2 < f(x) \le 3 \longrightarrow 3 - (x+1) \quad 1 \le x \le 2$$

$$g(x) = \begin{cases} x+2 & ; \quad 0 \le x \le 1 \\ 2-x & ; \quad 1 < x \le 2 \\ 4-x & ; \quad 2 < x \le 3 \end{cases}$$




Pt. of disc (s) are 1 & 2.

**Sol.10** $f(x) = \begin{cases} \dfrac{e^{\{(x+2)\ell n4\}^{\frac{[x+1]}{4}}} - 16}{4^x - 16} & ; \quad x < 2 \\ A\dfrac{1-\cos(x-2)}{(x-2)\tan(x-2)} & ; \quad x > 2 \end{cases}$

$$f(2^-) = \lim_{x\to 2^-} \frac{(4^{(x+2)})^{\frac{[x+1]}{4}} - 16}{4^{2-h} - 16}$$

$$= \lim_{h\to 0} \frac{(4^{(4-h)})^{\frac{2}{4}} - 16}{4^{2-h} - 16} \Rightarrow \lim_{h\to 0} \frac{4^{2-\frac{h}{2}} - 16}{4^{2-h} - 16}$$

$$\lim_{h\to 0} \frac{4^{2-\frac{h}{2}}\left\{1 - 4^{2-2+\frac{h}{2}}\right\}}{4^{2-h}\{1 - 4^{2-2+h}\}}$$

$$\lim_{h\to 0} 4^{\frac{h}{2}} \left\{\frac{1 - 4^{\frac{4}{2}}}{1 - 4^{h}}\right\}$$

$$\lim_{h\to 0} 4^{\frac{h}{2}} \left\{\frac{\dfrac{4^h - 1}{1} \times 4}{\dfrac{4^h - 1}{4} \times 4}\right\} = \frac{L}{2}$$

$$f(2^+) = \lim_{x\to 2^+} A \frac{1-\cos(x-2)}{(x-2)\tan(x-2)}$$

$$= \lim_{h\to 0} A \frac{(1-\cosh)}{\dfrac{h(\tanh)}{h} \times h} = \frac{A}{2}$$

If f(x) is cont. then, A = 1 & f(2) = $\dfrac{1}{2}$

**Sol.11** $f(x) = \begin{cases} \left(\dfrac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}} & 0 < x < \dfrac{\pi}{2} \\ b + 2 & x = \dfrac{\pi}{2} \\ (1 + |\cos x|)^{\frac{a|\tan x|}{b}} & \dfrac{\pi}{2} < x < \pi \end{cases}$

$$f\left(\frac{\pi^-}{2}\right) = \lim_{h\to 0}\left(\frac{6}{5}\right)^{\frac{\tan 6\left(\frac{\pi}{2}-h\right)}{\tan 5\left(\frac{\pi}{2}-h\right)}}$$

$$= \lim_{h\to 0}\left(\frac{6}{5}\right)^{\frac{\tan(3x-6h)}{\tan\left(\frac{5\pi}{2}-5h\right)}} = \lim_{h\to 0}\left(\frac{6}{5}\right)^{\frac{-\tan 6h}{\cot 5h}} = 1$$

$$f\left(\frac{\pi^+}{2}\right) = \lim_{x\to\frac{\pi^+}{2}}(1+\cos x)^{\frac{a(\tan x)}{b}}$$

$$\ell = \lim_{h\to 0} -\frac{a\cot h}{b}(-\sin h)$$

$$\ell = \lim_{h\to 0} + \frac{a.\cos h}{b}$$

$$\ell = \frac{a}{b} \Rightarrow e^{a/b}$$

$$f\left(\frac{\pi}{2}\right) = b + 2 \Rightarrow \boxed{b=1} \neq e^a = 1 \Rightarrow \boxed{a=0}$$

Hence $= f\left(\frac{\pi^-}{2}\right) = f\left(\frac{\pi}{2}\right)$

So, $\boxed{a=\frac{1}{2}}$

Now, $f\left(\frac{\pi^+}{2}\right) = \lim_{h\to 0}\frac{b(1-\cos h)}{4h^2} = \frac{b}{2\times 4}$

$\Rightarrow \boxed{b=4}$

**Sol.12** $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & ; \ x < \dfrac{\pi}{2} \\ a & ; \ x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & ; \ x > \dfrac{\pi}{2} \end{cases}$

$$f\left(\frac{\pi^-}{2}\right) = \lim_{x\to\frac{\pi^-}{2}}\frac{1-\sin^3 x}{3\cos^2 x}$$

$$= \lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$\Rightarrow \lim_{h\to 0}\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3\frac{\sin^2 h}{h^2}\times h^2}$$

$$= \frac{1}{6}\times(1+1+1) = \frac{1}{2}$$

$f\left(\frac{\pi}{2}\right) = a.$ since $f(x)$ is cont at $x = \frac{\pi}{2}$.

**Sol.13** $f(x) = \begin{cases} \dfrac{\sin(a+1)x+\sin x}{x} & ; \ x<0 \\ c & ; \ x=0 \\ \dfrac{(x+bx^2)^{\frac{1}{2}}-x^{\frac{1}{2}}}{bx^{3/2}} & ; \ x>0 \end{cases}$

$$f(0^-) = \lim_{h\to 0}\frac{\sin(a+1)(-h)-\sin h}{-h}$$

$$= \lim_{h\to 0}\frac{[\sin(a+1)h](a+1)}{h(a+1)} + \lim_{h\to 0}\frac{\sin h}{h}$$

$= a + 1 + 1 = \boxed{a+2}$

$f(0) = c,$

$$f(0^+) = \lim_{h\to 0}\frac{(h+bh^2)^{\frac{1}{2}}-h^{\frac{1}{2}}}{bh^{\frac{3}{2}}}$$

$$= \lim_{h\to 0}\frac{h+bh^2-h}{bh^{3/2}\{(h+bh^2)^{1/2}+h^{1/2}\}}$$

$$\Rightarrow \lim_{h\to 0}\frac{h^{1/2}}{\{h^{1/2}(1+bh)^{1/2}+1\}} = 1/2$$

Since 'f' is cont at x = 0, so,

$f(0^+) = f(0^-) = f(0)$

$\frac{1}{2} = a + 2 = c \Rightarrow \boxed{C=\frac{1}{2}}$

**Sol.14** $f(x) = \dfrac{\sin 3x + A\sin 2x + B\sin x}{x^5}$

$$= \lim_{x\to 0} \frac{\left\{3x - \frac{(3x)^3}{3} + \frac{(3x)^5}{5} + A\right\} + A\left\{2x - \frac{(2x)^3}{3} + \frac{(2x)^5}{5}\right\} + B\left\{x - \frac{x^3}{3} + \frac{x^5}{5}\right\}}{x^5}$$

$$\lim_{x\to 0} \frac{(3+2A+B)x}{x^5} - \frac{x^3\left(9 + \frac{8A}{3} + \frac{B}{3}\right)}{x^5}$$

$$+ \frac{x^5}{x^5}\left(\frac{3^5}{5} + \frac{32A}{5} + \frac{B}{5}\right)$$

For existence of limit,

$3 + 2A + B = 0$ & $9 + \frac{8A}{3} + \frac{B}{3} = 0$

$2A + B + 3 = 0$ & $27 + 8A + B = 0$

$6A + 24 = 0 \quad \Rightarrow \quad \boxed{A = -4}$

$\boxed{B = 5}$

$$f(0) = \frac{243}{5} - \frac{128}{5} + 1$$

$$f(0) = \frac{115}{5} + 1 \Rightarrow \quad \boxed{24}$$

**Sol.15**

$$f(x) = \begin{cases} \dfrac{\left\{\frac{\pi/2}{2} - \sin^{-1}(1-\{x\}^2)\right\}\sin^{-1}(1-\{X\})}{\sqrt{2}(\{x\}-\{x\}^3)} & , \ x \ne 0 \\ \dfrac{\pi}{2} & , \ x = 0 \end{cases}$$

$$g(x) = \begin{cases} f(x) & , \ x \ge 0 \\ 2\sqrt{2}f(x) & , \ x < 0 \end{cases}$$

$$f(0^-) = \lim_{h\to 0} \frac{\left\{\frac{\pi}{2} - \sin^{1}(1-(1-h)^2)\right\}\sin^{-1}(1-1-h)}{\sqrt{2}\{1-h-(1-h)^3\}}$$

$$= \lim_{h\to 0} \frac{\left\{\frac{\pi}{2} - \sin^{-1}(1-1-h^2+2h)\right\}\sin^{-1}h}{\sqrt{2}(1-h)(1-h^2-h-1)}$$

$$= \lim_{h\to 0} \frac{\left\{\frac{\pi}{2} - \sin^{-1}(2h-h^2)\right\}\sin^{-1}h}{\sqrt{2}(1-h)(1-h)(h+1)}$$

$$= -\lim_{h\to 0} \frac{\cos^{-1}(2h-h^2)}{\sqrt{2}(1-h^2)} = \frac{-\pi}{2\sqrt{2}}$$

$$f(0^+) = \lim_{h\to 0} \frac{\left\{\frac{\pi}{2} - \sin^{-1}(1-h^2)\right\}\left\{\sin^{-1}(1-h)\right\}}{\sqrt{2}(h-h^3)}$$

$$= \lim_{h\to 0} \left\{\frac{\cos^{-1}(1-h^2).\sin^{-1}(1-h)}{\sqrt{2}\,h(1+h)(1-h)}\right\}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(1-h)}{\sqrt{1-h^2}} = \frac{\pi}{2}$$

$$g(x) = \begin{cases} \dfrac{\left\{\frac{\pi}{2} - \sin^{-1}(1-\{x\}^2)\right\}\sin^{-1}(1-\{x\})}{\sqrt{2}(\{x\}-\{x\}^3)} & x \ge 0 \\ \end{cases}$$

**Sol.16** $f(x) = \begin{cases} |4x-5|[x] & x > 1 \\ [\cos \pi x] & x \le 1 \end{cases}$

Defining the given fn as follows :

$$f(x) = \begin{cases} 1 & x = 0 \\ 0 & \left(0, \frac{1}{2}\right) \\ -1 & \left(\frac{1}{2}, 1\right] \\ & \left(1, \frac{5}{4}\right) \\ & x = 2 \end{cases}$$

**Sol.17** $f(x) = x + \{-x\} + [x]$

$= x + (-x - [x]) + [x]$

$f(x) = [x] - [-x]$

$$f(x) = \begin{cases} -3 & -2 \le x < -1 \\ -1 & -1 \le x < 0 \\ 1 & 0 \le x < 1 \\ 3 & 1 \le x < 2 \\ 4 & x = 2 \end{cases}$$

f(x) discont. at every intiger value is [–2, 2]

**Sol.18** $f(x) = \begin{cases} 3x - b \ ; & x \le 1 \\ 3x \ ; & 1 < x < 2 \\ 5x^2 - a \ ; & x \ge 2 \end{cases}$

at x = 1

LHL = a – b ; RHL = 3

LHL = RHL

$\Rightarrow$ a – b = 3

Locus x – y = 3 .....(i)

at x = 2

LHL = 6 ; RHL = 4b – a

4b – a = 6

or 4y – x = 6 .....(ii)

from (i) & (ii)

y = 3

But y $\neq$ 3 because discont. at x= 2

**Sol.19** $f(x) = \lim_{n \to \infty} \frac{ax^2 + bx + c + (e^x)^n}{1 + c(e^x)^n}$

$$f(x) = \begin{cases} 1/c & ; \; e^x > 1 \Rightarrow x > 0 \\ \frac{ax^2 + bx + c + 1}{1 + c} = 1 & ; \; e^x = 1 \Rightarrow x > 0 \\ ax^2 + bx + c & ; \; e^x < 1 \Rightarrow x < 0 \end{cases}$$

For cont.

$f(0) = f(0^+) = f(0^-)$

$1 = \frac{1}{c} \Rightarrow c = 1$

a,b$\in$ R

**Sol.20** $g(x) = \lim_{n \to \infty} \frac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}$

$$g(x) = \begin{cases} \frac{f(x)}{2} & ; \; x > 1 \\ \frac{f(1) + h(1) + 1}{8} & ; \; x = 1 \\ \frac{h(x) + 1}{3x + 3} & ; \; x < 1 \end{cases}$$

$g(x) = \lim_{x \to 1} \frac{\text{Sin}^2(\pi 2^x)}{\ln(\text{Sec}(\pi - 2^x))}$

$= \lim_{x \to 1} \frac{-\text{Sin}^2(\pi 2^x)}{\ln \frac{(1 + \cos \pi 2^x - 1)}{\cos \pi 2^x - 1}} \times \frac{1}{\cos \pi . 2^x - 1}$

$= \lim_{x \to 1} \frac{\text{Sin}^2(2\pi - \pi 2^x)}{\left(\frac{1 - \cos \pi 2^x - 1}{(\pi . 2^x)}\right) \times (\pi 2^x)}$

g(1) = 2

$g(1) = \frac{f(1) + h(1) + 1}{8} = 2$

$\Rightarrow$ f(1) +h(1) =15 -----(i)

$g(1^+) = (1^-)$

$\frac{f(1)}{2} = \frac{h(1) + 1}{6} \Rightarrow$ 3fu) = hu) + 1 ----- (ii)

from (ii) & (i)

h (1) = 11 and f(1) = 4

4g(1) + 2f(1) – h(1)

4g(1) + 2(f(1) + h(1))–3h ( 1)

= 8 + 2 (15) – 3$\times$ 11 = 5

**Sol.21** $f(x) = \frac{2 + \cos x}{x^3 \sin x} - \frac{3}{x^4}$

$= \frac{2x + x\cos x - 3\sin x}{x^4 \sin x}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{2x + x\cos x - 3\sin x}{x^4 \sin x}$

$= \lim_{x \to 0} \frac{2x + x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - - - -\right) - 3\left(1 - \frac{x^3}{3!} + \frac{x5}{5!} - - - -\right)}{x^5}$

$\lim_{x \to 0} \frac{\left(\frac{4}{4!} - \frac{3}{5!}\right)x^5 + - - - -}{x^5}$

$f(0) = \frac{1}{60}$

**Sol.22** $f(x) \; \frac{a^{\tan x}(a^{\sin x - \tan x} - 1)}{-(\sin x - \tan x)}$

$= \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{a^{\tan x}(a^{\sin x - \tan x} - 1)}{-(\sin x - \tan x)}$

$= -\ln a = \ln\left(\frac{1}{a}\right)$-----(i)

LHL = $\lim_{x \to 0} f(x) \; \lim_{x \to 0} \frac{\ln(1 + x^2 + x^4)}{\sec x - \cos x}$

$= \lim_{x \to 0} \dfrac{\cos x}{(1-\cos x)(1+\cos x)} \times \dfrac{\ln(1+x^2+x^4)}{(x^2+x^4)} \times (x^2+x^4)$

$\lim_{x \to 0} \dfrac{x^2(1+x^2)}{(1-\cos x)(1+\cos x)} = 1$

LHL = RHL

$\ln\left(\dfrac{1}{a}\right) = 1 \Rightarrow a = \dfrac{1}{e}$

$g(x) = \ln\left(2 - \dfrac{x}{a}\right)\cot(x-a)$

$\lim_{x \to \frac{1}{e}} \ln\left(2 - \dfrac{x}{a}\right)\cot\left(x - \dfrac{1}{e}\right)$

$\lim_{x \to \frac{1}{e}} \dfrac{\ln(1+1-xe)}{(1-xe)} \times \dfrac{(1-xe)}{\left(x - \frac{1}{e}\right)}$

$\lim_{x \to \frac{1}{e}} g(n) = -e$

"If g(n) is continous

then $g\left(\dfrac{1}{e}\right) = -e$

**Sol.23** $f(x+y) = f(x) + f(y)$

put x = 0, y = 0

$\Rightarrow f(0) = 0$ ------(i)

Let x = a

$f(a^t) = \lim_{x \to a^+} f(x)$

$= \lim_{x \to 0} f(a+h)$

$= \lim_{x \to 0} f(a) + f(n) = f(a)$ -----(ii)

$f(a^-) = \lim_{x \to a^-} f(x)$

$= \lim_{h \to 0} f(a-h)$

$= \lim_{h \to 0} f(a) + f(-h)$

$f(a^-) = f(a) + f(-h)$---- (iii)

from (ii) & (iii)

$f(a^+) = f(a^-) = f(a)$

so f(x) is cont. at x = a

$\Rightarrow$ f(x) is cont. $\forall$ all $x \in R$

**Sol.24** $f(x) = \sum_{r=1}^{n} \tan\left(\dfrac{x}{2^r}\right) \sec\left(\dfrac{x}{2^{r-1}}\right)$ ; r, n $\in$ N

Let $T_r = \tan\left(\dfrac{x}{2^r}\right) \sec\left(\dfrac{x}{2^{r-1}}\right)$ $\theta = \dfrac{x}{2^r}$

$= \tan\theta \sec 2\theta$

$T_r = \dfrac{\sin\theta}{\cos\theta\cos 2\theta} = \dfrac{\sin(2\theta-\theta)}{\cos\theta\cos 2\theta}$

$= \tan 2\theta - \tan\theta$

$T_r = \tan\dfrac{x}{2^{r-1}} - \tan\dfrac{x}{2^r}$

$T_1 = \tan\dfrac{x}{2^0} - \tan\dfrac{x}{2}$

$T_2 = \tan\dfrac{x}{2} - \tan\dfrac{x}{2^2}$

$\vdots$

$T_n = \tan\dfrac{x}{2^{n-1}} - \tan\dfrac{x}{2^n}$

---

f(x) = sum of $T_r$

$f(x) = \tan x - \tan\dfrac{x}{2^n}$

$f(x) + \tan\dfrac{x}{2^n} = \tan x$

$g(x) = \lim_{n \to \infty} \dfrac{\ell n \tan x - (\tan x)^n \left[\sin\left(\tan\frac{x}{2}\right)\right]}{1 + (\tan x)^n}, x \neq \dfrac{\pi}{4}$

$g(x) = -\left[\sin\left(\tan\dfrac{x}{2}\right)\right]$ $\tan x > 1$

$\Rightarrow x > \dfrac{\pi}{4}$

$= \ell n(\tan x)$ $\tan x < 1$

$\Rightarrow x < \dfrac{\pi}{4}$

LHL $\left(\text{at } x = \frac{\pi}{4}\right) = 0$

RHL $\left(\text{at } x = \frac{\pi}{4}\right) = -\left[\sin\left(\tan\frac{\pi}{8}\right)\right]$

$= 0$

$\Rightarrow \quad k = 0$

g(x) = $\ell n\,(\tan x)$ if $0 < x < \frac{\pi}{4}$

$0$ if $\frac{\pi}{4} \le x < \frac{\pi}{2}$

g(x) is cont. everywhere is $\left(0, \frac{\pi}{2}\right)$

**Sol.25** $g(x) = k(x + 1)$

$h(x) = \dfrac{(x+1)(x^2-2x-1)}{k(x+1)}$

$\lim\limits_{x\to-1} h(x) = \frac{1}{2} \Rightarrow k = 4$

$g(x) = 4(x + 1)$

$h(0) = -1/4$

$f(0) = -1;\ \ g(0) = 4$

$\lim\limits_{x\to0} [3\,h(x) + f(x) - 2g(x)] = \dfrac{-39}{4}$

**Sol.26** **(a)** Let $f(x) = g(x) - x$

$f(a) = g(a) - a \le 0$

$f(b) = g(b) - b \ge 0$

$\exists \quad c \in [a, b] \Rightarrow f(c) = 0$

$\Rightarrow g(c) = c$

**(b)** Let $g(x) = f(x) - f\left(x + \frac{1}{2}\right)$

WWTPT $g(c) = 0 \quad \forall\ c \in \left[0, \frac{1}{2}\right]$

$x \in \left[0, \frac{1}{2}\right] \Rightarrow g(x)$ is cont.

$g(0) = f(0) - f\left(\frac{1}{2}\right)$

$g\left(\frac{1}{2}\right) = f\left(\frac{1}{2}\right) - f(1) = f\left(\frac{1}{2}\right) - f(1)$

$= -\left[f(0) - f\left(\frac{1}{2}\right)\right]$

so $g(0)$ & $g\left(\frac{1}{2}\right)$ are opposite signs.

so

$\exists\ c \in \left[0, \frac{1}{2}\right] \Rightarrow g(c) = 0$

$\Rightarrow f(c) = f\left(c + \frac{1}{2}\right)$

**Sol.27**

$g(0^-) = \lim\limits_{x\to0^-} \dfrac{1 - a^x + a^x\ \ell n\, a^x}{a^x . x^2}$ Let $a^x = t$

$= \ell n^2 a \lim\limits_{t\to1} \dfrac{1 - t + t\ \ell n\, t}{\ell n^2 t}$

$= \frac{1}{2}\ \ell n^2 a$

$g(0^+) = \lim\limits_{x\to0^+} \dfrac{(2a)^x - \ell n\,(2a)^x - 1}{\ell n^2\ t}$

$= \ell n^2 2a \lim\limits_{t\to1} \dfrac{t - \ell n\, t - 1}{\ell n^2 t} = \frac{1}{2}\ \ell n^2 2a$

$g(0^+) = g(0^-)$

$\frac{1}{2}\ \ell n^2 2a = \frac{1}{2}\ \ell n^2 a \Rightarrow a = \frac{1}{\sqrt{2}}$

$\Rightarrow g(0) = \frac{1}{2}\ \ell n^2 \frac{1}{\sqrt{2}} = \frac{1}{8}\ (\ell n\, 2)^2$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# EXERCISE – IV HINTS & SOLUTIONS

**Sol.1** Let $f_1(x) = \sin x$ & $f_2(x) = \sin |x|$
$f_1(x)$ is continuous & differentiable always.
$f_2(x)$ is continuous but not differentiable at $x = 0$.
so $f_1(x) + f_2(x) = f(x)$ is continious but not differentiable at $x = 0$.
(using fundamental theorems)

**Sol.2** $f(x) = \begin{cases} -3x+3 & x<0 \\ -x+3 & x\in[0,1) \\ x+1 & x\in[1,2) \\ 3x-3 & x>2 \end{cases}$

so continuous but non differentiable at $x = 0, 1, 2$.

**Sol.3** for differentiability

$$f'(0^+) = \lim_{h\to 0} \frac{h^n \sin 1/h}{h}$$

so only if $n \in (0, 1]$, function is non differentiable.

**Sol.4** $f'(0^+) = \lim_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to 0} \frac{1+\sin h - 1}{h} = 1$$

$f'(0^-) = \lim_{h\to 0} \dfrac{f(0-h)-f(0)}{h} = \lim_{h\to 0} \dfrac{0}{h} = 0$

so continuous but not differentiable at $x = 0$
(check at $x = \pi/2$)

**Sol.5** $f'(0^+) = \lim_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$$= \lim_{h\to 0} \frac{h.\tan^{-1}(1/h) - 0}{h}$$

$$= \frac{\pi}{2}$$

$f'(0^-) = \lim_{h\to 0} \dfrac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} -\tan^{-1}(1/h)$
$= -\pi/2$.
so continuous but non differentiable at $x = 0$.

**Sol.6** LHS $= \lim_{x\to 0} \int \dfrac{f(x)-f(0)}{x}$

$$+ \frac{1}{2}\left\{\frac{f(x/2)-f(0)}{x/2}\right\} + \ldots\ldots + \frac{1}{k}\left\{\frac{f(x/k)-f(0)}{x/k}\right\}$$

$$= f'(0) + \frac{f'(0)}{2} + \ldots\ldots + \frac{f'(0)}{k} = 1 + \frac{1}{2} + \ldots\ldots + \frac{1}{k}$$

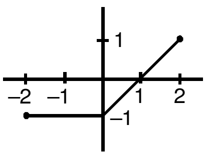
**Sol.7** $f'(0^+) = 0$
$f'(0^-) = 1$
so non differentiable but continuous at $x = 0$.

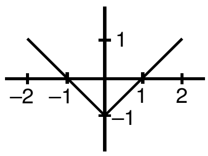
**Sol.8** $f'(1^+) = 3$ & $f'(1^-) = -1$

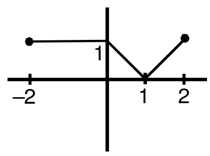
**Sol.9** If $f(x)$ is continuous then
$f(1^-) = f(1^+) \Rightarrow a(1) - b = -1$
$\Rightarrow \quad a - b = -1$
If $f(x)$ is differentiable then
$f'(1^+) = f'(1^-)$

$\Rightarrow \quad \dfrac{1}{x^2} = 2ax$ ; at $x = 1 \Rightarrow a = 1/2$

so $\quad b = 3/2$

**Sol.10** $f(x) =$ 


$f(|x|) =$ 


$|f(x)| =$ 


so check at 0 & 1 using fundamental theorem of addition.

**Sol.11** $f(x) = \begin{cases} 0 & ;\ x=-1 \\ \cos^{-1}\left(\text{sgn}\left(\dfrac{-4}{3x+2}\right)\right) = 0 & ;\ x<-1 \\ \cos^{-1}\left(\text{sgn}\left(\dfrac{-2}{3x+1}\right)\right) = 0 & ;\ x>-1 \end{cases}$

so continuous & differentiable at $x = -1$

$f(x) = \begin{cases} 0 & ;\ x=1 \\ 0 & ;\ x>1 \\ \pi/2 & ;\ x<1 \end{cases}$

so neither continuous non differentiable at $x = 1$.

**Sol.12** $f(x) = \begin{cases} 0 & ; 0 \le x < 1 \\ x & ; 1 \le x < 2 \\ 2(x-1) & ; 2 \le x < 3 \\ 3(x-1) & ; x = 3 \end{cases}$

check at x = 1 & x = 2.

**Sol.13** $f'(0^+) = \lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$

$= \lim\limits_{h \to 0} \dfrac{h.\dfrac{(e^h - 2)}{h} + 1}{h} = 1$

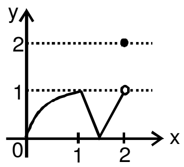
$f'(0^-) = \lim\limits_{h \to 0} \dfrac{f(0-h) - f(0)}{-h}$

$= \lim\limits_{h \to 0} \dfrac{\dfrac{(-h)[e^{-1-h} - 2]}{(-1-h)} - 2}{-h} = \text{ND.}$

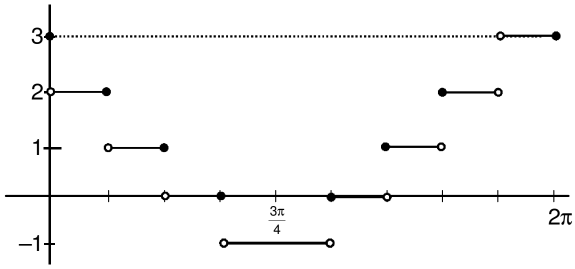
so non differentiable at x = 0

**Sol.14**

$f(x) = \begin{cases} \sin\dfrac{\pi x}{2} & ; x < 1 \\ 3 - 2x & ; [1, 3/2) \\ 2x - 3 & ; [3/2, 2) \\ 2 & ; x = 2 \end{cases} =$




**Sol.15**




$f(x) = \begin{cases} 3 \text{ if } \sin x = 0 \\ 2 \text{ if } -1/4 \le \sin x < 0 \\ 1 \text{ if } -1/2 \le \sin x < -1/4 \\ 0 \text{ if } -3/4 \le \sin x < -1/2 \\ -1 \text{ if } -1 \le \sin x < -3/4 \end{cases}$

$\Rightarrow$ sum = $12\pi = \dfrac{k\pi}{2} \Rightarrow k = 24$

**Sol.16** $f(1^-) = f(1^+) \Rightarrow b = 0$

& also $3p + q = 0$ ........(1)

$f'(3^-) = 1 = f'(3^+) = (2 \times 3)p + q$ ........(2)

from (1) & (2) $\Rightarrow p = 1/3, q = -1$.

also $f'(1^-) \ne f'(1^+)$

$\Rightarrow a \ne 1$

**Sol.17** $f'(0^+) = \lim\limits_{h \to 0} \dfrac{h.\dfrac{a^{1/h} - a^{-1/h}}{a^{1/h} + a^{-1/h}} - 0}{h}$

$= \dfrac{a^{1/h} - a^{-1/h}}{a^{1/h} + a^{-1/h}}$ $\begin{cases} 0, a = 1 \\ -1, a \in (0,1) \\ 1, a > 1 \end{cases}$

$f'(0^-) = \begin{cases} 1, a \in (0,1) \\ 0, a = 1 \\ -1, a > 1 \end{cases}$

so continuous but not differentiable for a > 1 & a < 1.

**Sol.18** at x = 0

$f(0^+) = \lim\limits_{h \to 0} \underset{\substack{\downarrow \\ 0}}{h} \underbrace{\sin 1/h \sin \dfrac{1}{h.\sin 1/h}}_{\text{finite}} = 0 = f(0^-) = f(0)$

$f\left(\dfrac{1}{r\pi^+}\right) = \lim\limits_{x \to \frac{1}{r\pi^+}} . \underset{\substack{\downarrow \\ 1/r\pi}}{x} \ \underset{\substack{\downarrow \\ 0}}{\sin 1/x} \underbrace{\sin\left(\dfrac{1}{x.\sin 1/x}\right)}_{b/w[-1,1]} = 0$

$= f\left(\dfrac{1^-}{r\pi}\right) = f\left(\dfrac{1}{r\pi}\right)$

so continious between [0, 1].

for differentiability :

$f'(0^+) = \lim\limits_{h \to 0} \dfrac{\left[\underbrace{h \sin\dfrac{1}{h}}_{\text{b/w }[-1,\,1]}.\sin\underbrace{\dfrac{1}{h.\sin 1/h}}_{\substack{\downarrow \\ 0}} - 0\right]}{h}$

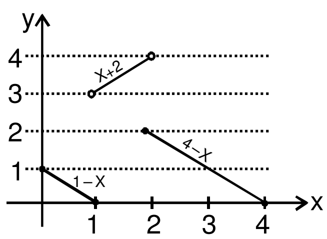
b/w [–1, 1]

= oscillating but finite

so $f'(0^+)$ = DNE

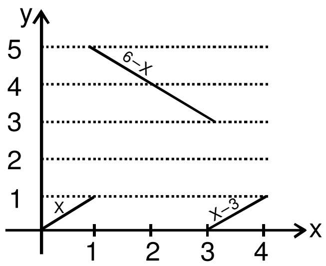
so non differentiable at x = 0.

**Sol.19** $f(x) \Rightarrow$

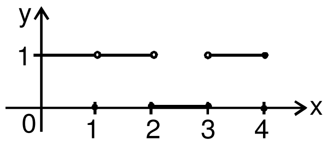



$f(f(x)) = \begin{cases} x & ; 0 < x < 1 \\ 6 - x & ; 1 < x < 3 \\ x - 3 & ; 3 < x < 4 \end{cases}$

f(f(x)) =




f[f(x)] ⇒




where [ * ] denotes GIF

**Sol.20** $f(0) = 1 \ ; \ f(0) > 0$

$$f'(x) = \lim_{h\to 0} \frac{f(x).f(h) - f(h)}{h}$$

$$= \lim_{h\to 0} f(x) . \underbrace{\frac{[f(h) - f(0)]}{h}}_{f'(0)} = -f(x).$$

**Sol.21** $f'(0^+) = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$

$$= \lim_{h\to 0} \frac{f(h) - f(kh) - f(o) + f(kh)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(kh)}{h} + \lim_{h\to 0} \frac{f(kh) - f(0)}{kh} \times k$$

$f'(0^+) = \alpha + k f'(0) \Rightarrow f'(0) = \alpha + k f'(0)$

$\Rightarrow \quad f'(0) = \dfrac{\alpha}{1 - k} \qquad\qquad f'(0^-) = \alpha + k f'(0)$

**Sol.22** $\lim_{x\to 0} \frac{f(x) - f(kx)}{x} = \alpha$

$$\Rightarrow \lim_{x\to 0} \frac{f'(x) - kf'(kx)}{1} = \alpha$$

$$\Rightarrow f'(0) - kf'(0) = \alpha \ \Rightarrow \ f'(0) = \frac{\alpha}{1-k}$$

$$f'(0^+) = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(kh) - f(0) + f(kh)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(kh)}{h} + \lim_{h\to 0} \left(\frac{f(kh) - f(0)}{kh}\right) \times k$$

$$= \alpha + k \, . \, f'(0) = \alpha + \frac{k\alpha}{1-k} = \frac{\alpha}{1-k}$$

$$f'(0^-) = \lim_{h\to 0} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{f(-h) - f(-kh) - f(0) + f(-kh)}{-h}$$

$$= \alpha + kf'(0) = \frac{\alpha}{1-k}$$

**Sol.23** $f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{h\to 0} \frac{h^2 \left|\cos \frac{\pi}{2h}\right| - 0}{h}$$

$$= \lim_{h\to 0} \underbrace{h}_{0} \underbrace{\left|\cos \frac{\pi}{2h}\right|}_{b/w\,[-1,1]} = 0 = f'(0^-)$$

so $f'(0) = 0$

$$f'\left(\frac{1^-}{3}\right) = -\frac{\pi}{2} \text{ and } f'\left(\frac{1^+}{3}\right) = \frac{\pi}{2}$$

$f'(x)$ fails to exist at $\dfrac{1}{2n+1}$ where $n \in I$.

**Sol.24** $f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{h\to 0} \frac{f[(h^{1/n})] - 0}{h} = \lim_{h\to 0} \frac{f((h^{1/n}))^n}{(h^{1/n})^n} = (f'(0))^n$$

$\Rightarrow f'(0)\,[f'(0)^{n-1} - 1] = 0 \ \Rightarrow f'(0) = 0$ or $\pm 1$
but $f'(0) \ge 0 \ \Rightarrow \ f'(0) = 0$ or 1.

Now $f'(x) = \lim_{h\to 0} \frac{f(x + (h^{1/n})^n) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{f(h^{1/n})^n}{(h^{1/n})^n} = (f'(0))^n$$

If $f'(0) = 0$ then $f'(x) = 0$ so $f'(0) \neq 0$
If $f'(0) = 1 \ \Rightarrow \ f(x) = x + c$
(using boundry condn c = 0)
$\Rightarrow \quad f(x) = x$ so $\quad f(10) = 10$
$f(x) - f(y) \ge \ell n\, x - \ell n\, y + x - y$
put $\quad x = x + h, \ y = x$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\ell n(x+h) - \ell n x + h}{h}$$

$$\Rightarrow f'(x) \ge -\frac{1}{x} + 1$$

If $x = x - h$ & $y = x$

$$\lim_{h\to 0} \frac{f(x-h) - f(x)}{-h} \ge \lim_{h\to 0} \frac{f(x-h) - \ell n\, x - h}{-h}$$

$$\Rightarrow f'(x) \ge \frac{1}{x} + 1 = g(x) \Rightarrow \sum_{n=1}^{\infty} g\left(\frac{1}{n}\right) = \sum_{n=1}^{\infty} (n+1) = 5150$$

# EXERCISE – V HINTS & SOLUTIONS

**Sol.1** $f(x) = [x]^2 - [x^2]$

$$\left.\begin{matrix}RHL = 0\\ LHL = 1\end{matrix}\right\} \text{ at } x = 0 \qquad \left.\begin{matrix}RHL = 0\\ LHL = 0\end{matrix}\right\} \text{ at } x = 1$$

**Sol.2** RHL $= \lim\limits_{x\to 0} \dfrac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1}$ ; $\dfrac{c^{1/3}-1}{0}$ form

$\Rightarrow c = 1$

$$= \lim_{x\to 0} \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1}$$

$$= \lim_{x\to 0} \frac{1+\frac{1}{3}x-1}{1+\frac{1}{2}x-1} = \frac{2}{3} \Rightarrow \quad b = \frac{2}{3}$$

LHL $= \lim\limits_{x\to 0} (1 + ax)^{1/x} = e^{\lim\limits_{x\to 0} a} = e^a$

$$e^a = \frac{2}{3} \Rightarrow a = \ell n\frac{2}{3}$$

$$a = \ell n\frac{2}{3} \ ; b = \frac{2}{3} \ ; c = 1$$

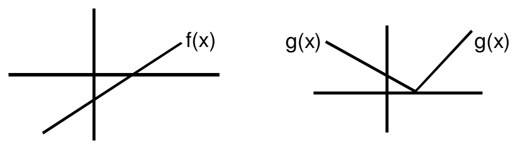
**Sol.3** RHL $\lim\limits_{x\to 1^+} \dfrac{e^{1/x-1}-2}{e^{1/x-1}+2} = \dfrac{1-0}{1+0} = 1$

LHL $\lim\limits_{x\to 1^-} \dfrac{e^{1/x-1}-2}{e^{1/x-1}+2} = \dfrac{0-2}{0+2} = -1$

discont at x = 1

**Sol.4** $f : R \to R$ $\qquad$ $g : R \to R$

$g(x) = |f(x)|$




**Sol.5** $f(x) = \dfrac{x}{1+|x|}$ ; $x \geq 1$ or $x \leq -1$

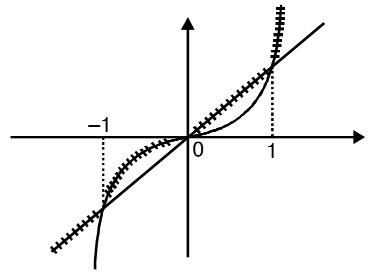
$$= \frac{x}{1-|x|}, -1 < x < 1$$

$f(x) = \dfrac{x}{1+x}$ ; $x \geq 1, \ -1 < x < 0$

$$= \frac{x}{1-x} \ ; x \leq -1, 0 \leq x < 1$$

Discont. at x = 1 and x = –1 hence not differentiable at x = 1, –1 and cont. & derivaiable at x = 0

**Sol.6** **(a)** $f : R \to R$ $\quad f(x) = \max [x, x^3]$




Non- diff at x = 0, 1, –1

**(b)** $f(x) = [x] \sin \pi x$

If x is just less than k, $[x] = k - 1$

$f(x) = (k - 1) \sin \pi x$,

LHD of f(x) $= \lim\limits_{x\to k} \dfrac{(k-1)\sin \pi x - k \sin \pi k}{x-k}$

$$= \lim_{h\to 0} \frac{(k-1)\sin \pi(k-h)}{-h} \qquad (x = k - h)$$

$$= \lim_{h\to 0} \frac{(k-1)(-1)^{k-1}\sin h\pi}{-h} = (-1)^k (k-1)\pi$$

**(c)** RHD of $\sin(|x|) - |x| = \lim\limits_{h\to 0} \dfrac{\sinh - h}{h} = 1 - 1 = 0$

$(\because f(0) = 0)$

LHD of $\sin(|x|) - |x|$

$$= \lim_{h\to 0} \frac{\sin|-h| - |-h|}{-h} = \frac{\sinh - h}{-h} = 0$$

**Sol.7** If $\lim\limits_{x\to \alpha^+} g(x) = \lim\limits_{x\to \alpha^-} g(x) = g(\alpha)$ &

$f(x) - f(\alpha) = g(x)(x - \alpha)$ ........(1)

so $f(x) - f(\alpha) = g(x)(x - \alpha)$ & $g(x)$ must be continous as $f'(\alpha^+) = f'(\alpha^-)$

**Sol.8** $f(x) = \tan^{-1} x$ $\qquad -1 \leq x \leq 1$

$= \dfrac{1}{2}(x - 1)$ $\qquad x > 1$

$= \dfrac{1}{2}(-x - 1)$ $\qquad x < -1$

f(x) is discontinuous at x = 1, –1 hence non-diff. at x = 1, –1

**Sol.9** $f : R \to R$ $f(1) = 3$ ; $f'(1) = 6$

$$\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{1/x} = e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{f(1+x)-f(1)}{f(1)}\right)}$$

$$= e^{\frac{f'(1)}{f(1)}} = e^2$$

**Sol.10** $f(x) = \begin{cases} x+a\ ,\ x<0 \\ x-1\ ,\ x\ge 1 \\ 1-x\ ,\ 0\le x<1 \end{cases}$ ; $g(x) = \begin{cases} x+1\ ,\ x<0 \\ (x-1)^2+b\ ,\ x\ge 0 \end{cases}$

$gof(x) = g[f(x)] = f(x) + 1$ , $f(x) < 0$

$[f(x) - 1]^2 + b$ , $f(x) \ge 0$

Now, $f(x) < 0$

$$\Rightarrow \begin{cases} x+a<0\ ,\ x<0 \\ x-1<0\ ,\ x\ge 1 \\ 1-x<0\ ,\ 0\le x<1 \end{cases}$$

$\Rightarrow x < -a$ when $x < 0$

$x < 1$ when $x \ge 1$

$x > 1$ when $0 \le x < 1$

The last two cases are not possible

so, $f(x) < 0$ if $x < -a$

a is positive

$f(x) < 0$ for $x < -a$

$\Rightarrow f(x) \ge 0$ for $x > -a$

Now,

$$gof(x) = \begin{cases} f(x)+1\ ,\ x<-a \\ [f(x)-1]^2+b\ ,\ x\ge -a \end{cases} \text{ where } f(x) = x + a$$

$$gof(x) = \begin{cases} x+a+1\ ,\ x<-a \\ (x+a-1)^2+b\ ,\ -a\le x<0 \end{cases}$$

$= (1 - x - 1)^2 + b$, $0 \le x < 1$

$= x^2 + b$, $0 \le x < 1$

$gof(x) = (x - 1 - 1)^2 + b$, $x \ge 1$

$= (x - 2)^2 + b$ , $x \ge 1$

since, gof is continuous for all real x, therefore,

$(a - 1)^2 + b = b \Rightarrow a = 1$, b is any real number.

for $a = 1$, $b \in R$, gof is continuous

$$gof(x) = \begin{cases} x+2\ ,\ x<-a \\ x^2+b\ ,\ -a\le x<1 \\ (x-2)^2+b\ ,\ x\ge 1 \end{cases}$$

so gof is differentiable at $x = 0$ if $a = 1$, $b \in R$

**Sol.11** $f'(a^-) = \lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h} = 0,\ x \in (0, 2a)$

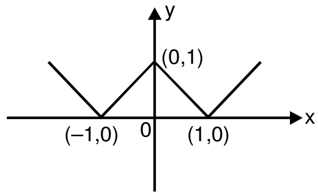
Now $f'(-a^-) = \lim_{h\to 0}\dfrac{f(-a+h)-f(-a)}{h}$

$= \lim_{h\to 0}\dfrac{-f(a-h)+f(a)}{h}$, f is an odd function

$= \lim_{h\to 0}\dfrac{-f(a+h)+f(a)}{h}$ ; $f(x) = f(2a - x), x \in (a, 2a)$

$= -\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h} = 0$

**Sol.12** **(a)** $y = ||x| - 1|$




Non-differentiable at $x = 1, 0, -1$

**(b)** $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$

$\Rightarrow \lim_{x_1\to x_2}\left|\dfrac{f(x_1)-f(x_2)}{x_1-x_2}\right| < \lim_{x_1\to x_2}|x_1 - x_2|$

$\Rightarrow |f'(x)| < \delta \Rightarrow f'(x) = 0$

Hence f(x) is a constant function and P(1, 2) lies on the course.

$\Rightarrow f(x) = 2$ is the curve.

Hence the equation of tangent is $y - 2 = 0$

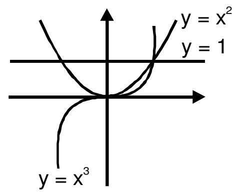
**Sol.13** $f(x) = \min. \{1, x^2, x^3\}$

$f(x) = x^3$ , $x \le 1$

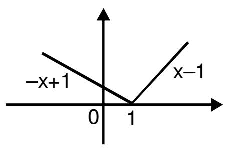
$1$ , $x > 1$

f(x) is continuous $\forall\ x \in R$

and non-diff. at $x = 1$




**Sol.14** From graph, $p = -1$




$\Rightarrow \lim_{x\to 1^+} g(x) = -1$

$\Rightarrow \lim_{h\to 0} g(1 + h) = -1$

$\Rightarrow \lim_{h\to 0}\left(\dfrac{h^n}{\log\cos^m h}\right) = -1$

$$\Rightarrow \lim_{h\to 0} \frac{n.h^{n-1}}{m.(-\tan h)} = \left(-\frac{n}{m}\right) \lim_{h\to 0} \left(\frac{h^{n-1}}{\tan h}\right) = -1$$

which holds if n = m = 2

**Sol.15** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{f(x)+f(h)-f(x)}{h} = f'(0)$

$\{\because f(0)=0\}$ = k(let)

$\Rightarrow$ f(x) = kx + c ; c = 0

hence f(x) = kx

**Sol.16** Let $y = \frac{b-x}{1-bx} \Rightarrow x = \frac{b-y}{1-by}$

so $f^{-1}(x) = \frac{b-x}{1-bx}$

so $f = f^{-1}$ on (0, 1)

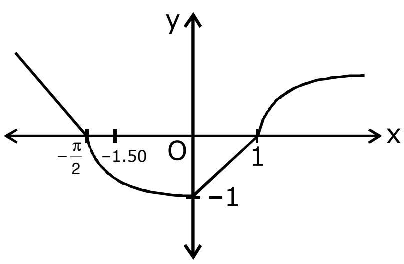
$f'(0) = 1 - b^2$ & $f'(b) = \frac{1}{1-b^2}$

so $f'(b) = \frac{1}{f'(0)}$

also $f^{-1}$ is differentiable on (0, 1)

**Sol.17**

graph of f(x) :



## Answer Ex–I SINGLE CORRECT (OBJECTIVE QUESTIONS)

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** A | **2.** B | **3.** D | **4.** C | **5.** C | **6.** B | **7.** B |
| **8.** B | **9.** B | **10.** B | **11.** B | **12.** D | **13.** B | **14.** D |
| **15.** D | **16.** D | **17.** C | **18.** B | **19.** D | **20.** C | **21.** D |
| **22.** B | **23.** B | **24.** C | **25.** C | **26.** C | **27.** C | **28.** C |
| **29.** C | **30.** D | **31.** A | **32.** D | **33.** C | **34.** D | **35.** D |
| **36.** B | **37.** B | **38.** C | **39.** C | **40.** B | **41.** D | **42.** D |
| **43.** D | **44.** B | **45.** C | **46.** D | **47.** A | **48.** B | **49.** C |
| **50.** B | **51.** B | **52.** C | **53.** A | **54.** B | **55.** A | **56.** D |
| **57.** A | **58.** D | **59.** C | **60.** B | **61.** A | **62.** D | **63.** D |
| **64.** D | **65.** D | | | | | |

## Answer Ex–II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** ABC | **2.** BCD | **3.** ABC | **4.** ABD | **5.** ABC | **6.** BD | **7.** ABC |
| **8.** AC | **9.** ABD | **10.** BD | **11.** ABD | **12.** ABD | **13.** AC | **14.** B, C |
| **15.** C, D | **16.** AC | **17.** AB | **18.** BD | **19.** BD | **20.** ABC | **21.** BD |
| **22.** ABD | **23.** ABD | **24.** ABD | | | | |

## Answer Ex–III SUBJECTIVE QUESTIONS (CONTINUITY)

**1.** –1 **2.** a = 0, b = 1 **3.** $f(0^+) = -2$ ; $f(0^-) = 2$ hence f(0) not possible to define

**4. (a)** –2, 2, 3 **(b)** K = 5 **(c)** even **5.** $y_n(x)$ is continuous at x = 0 for all n and y(x) is dicontinuous at x = 0

**6.** f is cont. in $-1 \le x \le 1$ **7.** P not possible. **8. (a)** $4 - 3\sqrt{2} + a$, **(b)** a = 3

**9.** g(x) = 2 + x for $0 \le x \le 1$, 2 – x for $1 < x \le 2$, 4 – x for $2 < x \le 3$, g is discontinuous at x = 1 & x = 2

**10.** A = 1 ; f(2) = 1/2 **11.** a = 0 ; b = –1 **12.** a = 1/2, b = 4 **13.** a = – 3/2, b ≠ 0, c = 1/2

**14.** A = –4, B = 5, f(0) = 1 **15.** $f(0^+) = \frac{\pi}{2}$ ; $f(0^-) = \frac{\pi}{4\sqrt{2}}$ $\Rightarrow$ f is discont. at x = 0 ;

$g(0^+) = g(0^-) = g(0) = \pi/2 \Rightarrow$ g is cont. at x = 0

**16.** the function f is continuous everywhere in [0, 2] except for x = 0, $\frac{1}{2}$, 1 & 2

**17**. discontinuous at all integral values in [–2, 2]

**18.** locus $(a, b) \to x, y$ is y = x – 3 excluding the points where y = 3 intersects it.

**19.** c = 1, a, b ∈ R **20.** 5 **21.** $\frac{1}{60}$

**24.** k = 0 ; g(x) = $\begin{cases} \ell n(\tan x) & \text{if } 0 < x < \frac{\pi}{4} \\ 0 & \text{if } \frac{\pi}{4} \le x < \frac{\pi}{2} \end{cases}$. Hence g(x) is continuous everywhere.

**25.** g(x) = 4(x + 1) and limit = $-\frac{39}{4}$

**27.** a = $\frac{1}{\sqrt{2}}$, g(0) = $\frac{(\ell n\, 2)^2}{8}$

## Answer Ex–IV SUBJECTIVE QUESTIONS (DIFFERENTIABILITY)

**1.** f(x) is conti. but not derivable at x = 0

**2.** conti. $\forall$ $x \in R$, not diff. at x = 0, 1 & 2

**3.** $0 < n \le 1$

**4.** conti. but not diff. at x = 0; diff. & conti. at x = $\pi/2$

**5.** conti. but not diff. at x = 0

**7.** f is cont. but not diff. at x = 0

**8.** $f'(1^+) = 3$, $f'(1^-) = -1$

**9.** a = 1/2, b = 3/2

**10.** not derivable at x = 0 & x = 1

**11.** f is cont. & derivable at x = –1 but f is neither cont. nor derivable at x = 1

**12.** discontinuous & not derivable at x = 1, continuous but not derivable at x = 2

**13.** not derivable at x = 0

**14.** f is conti. at x = 1, 3/2 & disconti. at x = 2, f is not diff. at x = 1, 3/2, 2

**15.** 24

**16.** $a \ne 1$, b = 0, p = $\frac{1}{3}$ and q = – 1

**17.** If $a \in (0, 1)$ $f'(0^+) = -1$; $f'(0^-) = 1$ $\Rightarrow$ continuous but not derivable

If a = 1 ; f(x) = 0 which is constant $\Rightarrow$ continuous but not derivable

If a > 1 $f'(0^-) = -1$ ; $f'(0^+) = 1$ $\Rightarrow$ continuous but not derivable

**18.** conti. in $0 \le x \le 1$ & not diff. at x = 0

**19.** f is conti. but not diff. at x = 1, disconti. at x = 2 & x = 3. cont. & diff. at all other points

**20.** $f'(x) = -f(x)$

**21.** continuous but not derivable at x = $\sqrt{2}$

**22.** $f'(0) = \frac{\alpha}{1-k}$

**23.** **(a)** $f'(0) = 0$, **(b)** $f'\left(\frac{1^-}{3}\right) = -\frac{\pi}{2}$ and $f'\left(\frac{1^+}{3}\right) = \frac{\pi}{2}$, **(c)** x = $\frac{1}{2n+1}$ $n \in I$

**24.** f(x) = x $\Rightarrow$ f(10) = 10

**25.** 5150

## Answer Ex–V JEE PROBLEMS

**1.** D

**2.** a = $ln\frac{2}{3}$ ; b = $\frac{2}{3}$ ; c = 1

**3.** Discontinuous at x = 1; $f(1^+) = 1$ and $f(1^-) = -1$

**4.** C

**5.** Discont. Hence hot deri. at x = 1 & –1. Cont. & deri. at x = 0

**6.** **(a)** D, **(b)** A, **(c)** D

**8.** D

**9.** C

**10.** a = 1 ; b = 0 $(gof)'(0) = 0$

**11.** $f'(a^-) = 0$

**12.** **(a)** A, **(b)** y – 2 = 0

**13.** A, C

**14.** C

**15.** B,C

**16.** C,D

**17.** A,B,C,D